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The Parameter Averaging Technique in Finite-Difference Modeling of Elastic Waves in Combined Structures with Solid, Fluid and Porous Subregions

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Abstract. To finite-difference model elastic wave propagation in a combined structure with solid, fluid and porous subregions, a set of modified Biot's equations are used, which can be reduced to the governing equations in solids, fluids as well as fluid-saturated porous media. Based on the modified Biot's equations, the field quantities are finite-difference discretized into unified forms in the whole structure, including those on any interface between the solid, fluid and porous subregions. For the discrete equations on interfaces, however, the harmonic mean of shear modulus and the arithmetic mean of the other parameters on both sides of the interfaces are used. These parameter averaging equations are validated by deriving from the continuity conditions on the interfaces. As an example of using the parameter averaging technique, a 2-D finite-difference scheme with a velocity-stress staggered grid in cylindrical coordinates is implemented to simulate the acoustic logs in porous formations. The finite-difference simulations of the acoustic logging in a homogeneous formation agree well with those obtained by the analytical method. The acoustic logs with mud cakes clinging to the borehole well are simulated for investigating the effect of mud cake on the acoustic logs. The acoustic logs with a varying radius borehole embedded in a horizontally stratified formation are also simulated by using the proposed finite-difference scheme.

AMS subject classifications: 76M20, 65M06, 76S05, 35L05, 86A15

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1 Introduction

In petroleum exploration a reservoir can be described as a fluid-saturated porous medium, which consists of solid frame and pore fluid. Studies of elastic wave propagation in

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such a medium become increasingly important to obtain more detailed reservoir information and monitor pore fluid flow. Biot [3–5] put forward the theory of wave propagation in a homogeneous fluid-saturated porous medium allowed for the interaction between the solid frame and the pore fluid. An important achievement of Biot's theory was the prediction of the second kind compressional wave in one-fluid-saturated porous media [19]. After the establishment of Biot's theory, many investigators have used it to study the wave propagation in porous formations. For example, Deresiewicz and Skalak [10] provided the boundary conditions on an interface between two different porous media. Rosenbaum [22] and Schmitt [23] calculated the monopole acoustic logs in homogeneous porous formations by applying the real-axis integration (RAI) method. Norris [18] derived the analytical fundamental solutions in the form of Green's function for a point force applied in an unbounded poroelastic medium. Based on the Green's function, the wave propagation in horizontally stratified porous formations was simulated by Boutin et al. [7]. However, analytical solutions for Biot's equations are in general impossible for arbitrary heterogeneous porous formations. In order to simulate poroelastic wave propagation in complex cases, finite-difference methods were applied (e.g., Zhu and McMechan [31], Dai et al. [9], Zhang [29], Zeng et al. [30], Song et al. [24], and Masson et al. [16]).

In a finite-difference algorithm, the heterogeneous formation is divided into discrete grid, so that the field quantities and governing equations for wave propagation are discretized to the grid. Due to the parameter discontinuities across an interface between different homogeneous media, the discrete equations for field quantities on the interface are commonly different from those for the ones in homogeneous media. Masson et al. [16] directly formulated the discrete equations on an interface between two different porous media by replacing the parameters in the discrete equations in homogeneous media with the averages of the two porous media. This parameter averaging technique is originated from the finite-difference modeling of electromagnetic waves. It has been derived that the discrete equations with average parameters comply with the continuity conditions of the electromagnetic fields on the interface [14]. For the finite-difference modeling of elastic wave propagation, however, the parameter averaging technique has not been validated by derivation in literature. Thus in previous papers the parameters in the discrete equations on interfaces were averaged by different manners. For example, the harmonic mean of shear modulus was used in Masson et al. [16] and in Song [25], while the arithmetic mean was used in Mittet [17]. Moreover, the parameter averaging technique was used only for interfaces between two media of the same type, such as porous-porous and solid-solid interfaces. If the wave equations on both sides of the interfaces are in different forms, such as for a fluid-porous interface, the parameter averaging technique cannot be used. Dong et al. [11] and Guan et al. [13] respectively formulated the discrete equations on fluid-porous interface by solving the linear equations based on the continuity conditions across the interface. Nevertheless, their method complicates the algorithm implementation because of the additional deriving on various interfaces at different locations, and is inconvenient for the intersection between three different media.