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IMEX Evolution of Scalar Fields on Curved Backgrounds

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Abstract. Inspiral of binary black holes occurs over a time-scale of many orbits, far longer than the dynamical time-scale of the individual black holes. Explicit evolutions of a binary system therefore require excessively many time-steps to capture interesting dynamics. We present a strategy to overcome the Courant-Friedrichs-Lewy condition in such evolutions, one relying on modern implicit-explicit ODE solvers and multidomain spectral methods for elliptic equations. Our analysis considers the model problem of a forced scalar field propagating on a generic curved background. Nevertheless, we encounter and address a number of issues pertinent to the binary black hole problem in full general relativity. Specializing to the Schwarzschild geometry in Kerr-Schild coordinates, we document the results of several numerical experiments testing our strategy.

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1 Introduction

Numerical simulations of the inspiral and merger of binary black holes (BBH) investigate Einstein's equations in the nonlinear regime where analytical progress often proves intractable. The primary goal of these simulations is the computation of gravitational waveforms necessary to analyze output from gravitational wave detectors like the "Laser

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Interferometric Gravitational Wave Observatory" (LIGO). Breakthroughs in 2005 have yielded two ways to simulate BBH evolutions: the *generalized harmonic system* (GHS) with excision [1] and the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) system with *moving punctures* [2,3]. Over the last few years numerical relativity has seen rapid progress along both fronts.

The evolution of a binary black hole proceeds through three phases. During the *inspiral* phase, the two separate black holes orbit about each other, with the orbit gradually tightening due to emission of angular momentum and energy via gravitational radiation. At small separation, the black holes encounter a dynamical instability, plunge rapidly toward each other and merge. This *merger* phase results in a single, larger, highly distorted black hole which subsequently relaxes to a stationary black hole during the *ringdown* phase. Merger and ringdown happen quickly, together lasting about $200M$, where the black hole mass M sets both the spatial and temporal scales. Therefore, merger and ringdown are comparatively easy to simulate at modest computational cost. In contrast, simulation of the inspiral phase is a daunting computational challenge. Because the orbital period increases rapidly with separation of the black holes, simulation of even a modest number of orbits requires much longer evolutions. For example, the last 10 orbits of an equal mass non-spinning binary black hole last about $2000M$, already an order of magnitude longer than merger and ringdown. Beyond necessarily longer time-spans, inspiral simulations also require higher accuracy. Indeed, gravitational wave flux decreases with separation, and it must be accurately resolved in order to compute the correct phasing of the gravitational waves.

To date all binary black hole simulations have employed explicit time-stepping, generally the method of lines with an explicit ODE scheme like the classical fourth-order Runge-Kutta method. Without question explicit time-stepping is appropriate for both merger and ringdown. However, during the inspiral phase, the relevant physical time-scale on which the binary separation changes is much longer than the dynamical time-scale M of each black hole. Nevertheless, the Courant condition associated with an explicit time-stepper heuristically requires that time-steps are proportional to the smallest grid spacing, and therefore explicit binary evolutions use time-steps that are typically of the order $M/100$ to $M/10$. For instance, a recent 16 orbit simulation [4] required nearly 200,000 explicit time-steps. This issue becomes more pronounced when modeling black holes with unequal masses $M_1 > M_2$. The orbital period is proportional to the total mass $M = M_1 + M_2$, whereas the Courant limit dictates that the time-step is proportional to the smaller mass M_2. The number of explicit time-steps needed to ensure numerical stability then scales like M/M_2. Due to these reasons, only a few binary black hole simulations with mass-ratios above 4:1 have been performed [5,6], and these are quite short and computationally expensive. Courant limitations are likewise more severe for simulations of spinning black holes, which also require higher spatial resolution close to the black holes.

These arguments suggest that some form of implicit time-stepping would more efficiently treat the inspiral phase, and this paper begins the study of alternative ways to carry out temporal integration of orbiting binaries in the early phase of their evolution.