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Finite-Element Analysis of Three-Dimensional Axisymmetrical Invisibility Cloaks and Other Metamaterial Devices

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Abstract. Accurate simulations of metamaterial devices are very important in the analysis of their electromagnetic properties. However, it is very difficult to make full-wave simulations of three-dimensional (3D) metamaterial devices due to the huge memory requirements and long computing time. In this paper, we present an efficient finite-element method (FEM) to analyze 3D axisymmetric electromagnetic devices designed by the transformation-optics approach, such as invisibility cloaks and concentrators. In the proposed method, we use the edge-based vector basis functions to expand the transverse field components, and the node-based scalar basis functions to expand the angular component. The FEM mesh is truncated with a cylindrical perfectly matched layer. We have applied the method to investigate the scattering from spherical and ellipsoidal invisibility cloaks and circularly cylindrical concentrators, in which the permittivity and permeability are both inhomogeneous and anisotropic. Numerical results are presented to show the validity and efficiency of the method.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: Finite-element method, axisymmetrical scatterer, metamaterial devices.

1 Introduction

Recently, invisibility cloaks based on metamaterials have aroused great interest [1–5]. Pendry et al. first proposed a coordinate transformation approach to provide a new method to control electromagnetic (EM) fields, by which a space consisting of the free

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space can be squeezed into a new space with different volume and space-distributed constitutive parameters [1]. Following this approach, a two-dimensional (2D) microwave invisibility cloak was experimentally realized [2], and some other metamaterial devices, such as the EM-wave concentrators [6,7], rotators [8], and hyperlens [9,10] have also been investigated by similar methods.

Besides experiments, the numerical simulation is the other important approach to analyze the EM properties of metamaterial devices. So far, several methods have been used to simulate the invisibility cloaks. The ray-tracing simulations [1,2] supporting the conclusions by Pendry et al. [1] were reported in the geometric optics limit. The full-wave finite-element simulations [3] were performed to study the effects of cloaking material perturbations to the propagation of the incident waves in a 2D cylindrical case. A rigorous solution to Maxwell's equations for a spherical cloak has been reported in [5]. The discrete dipole approximation method has been applied to simulate three-dimensional (3D) spherical cloaks and irregular 3D cloaks approximately [11,12]. However, the full-wave analysis and simulations of complicated 3D cloaks are still limited due to the large amount of computational burden and memory requirements. For example, the commercial software, COMSOL Multiphysics, which has been widely used in 2D situations, cannot be used to analyze large 3D cloaks and other metamaterial devices since the huge memory requirements and computational time.

The finite-element method (FEM) is characterized by the very flexible material handling capabilities and is often preferred for problems involving complex structures and inhomogeneous anisotropic materials [13,14]. By taking advantage of the rotational symmetry, the 3D problem can be reduced to a 2D computational domain. In this paper, an efficient FEM is proposed and applied to investigate the scattering from 3D axisymmetrical metamaterial devices designed by the transformation-optics approach, such as the invisibility cloaks and concentrators, whose permittivity and permeability are both inhomogeneous and anisotropic. In the proposed method, we use the edge-based vector basis functions to expand the transverse field components and the node-based scalar basis functions to expand the angular component, which automatically constrains the tangential field components to be continuous and eliminate the problem of spurious solutions without the need for a penalty factor. Triangular elements are applied to conveniently and accurately model arbitrary shapes of body of revolution. The FEM mesh is truncated with a cylindrical perfectly matched layer (PML) [13–15], which is much more efficient for arbitrarily-shaped scatterers than is a spherical boundary. The accurately numerical results are presented to show the validity and efficiency of the method.

2 FEM formulations for 3D axisymmetrical metamaterial devices

A metamaterial device with the PML enclosure is shown in Fig. 1. It is assumed here that the symmetric axis of axisymmetric metamaterial device with inhomogeneous and