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Test-Particle Simulation of Space Plasmas

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Abstract. Test-particle simulations provide a useful complement to the kinetic simulations of many-body systems and their approximate treatment with multiple moments. In a kinetic approach, systems are described at a microscopic level in terms of a large number of degrees of freedom. Fluid or multiple moment approaches, however, provide a description at the macroscopic level, in terms of relatively few physical parameters involving averages or moments of particle distribution functions. Ideally, fully kinetic descriptions should be done whenever possible. Due to their complexity, the use of these approaches is often not practical in many cases of interest. In comparison, the fluid approximation is much simpler to implement and solve. It can be used to describe complex phenomena in multi-dimensional geometry with realistic boundary conditions. Its main drawback is its inability to account for many phenomena taking place on fine space or time scales, or phenomena involving nonlocal transport. Macroscopic approaches are also not adapted to describe large deviations from local equilibrium, such as the occurrence of particle beams or otherwise strong anisotropy. With the test-particle method, particle trajectories are calculated using approximated fields obtained from a low level approach, such as multiple moments. Approximate fields can also be obtained from experiments or observations. Assuming that these fields are representative of actual systems, various kinetic and statistical properties of the system can then be calculated, such as particle distribution functions and moments thereof. In this paper, the test-particle method is discussed in the context of classical statistical physics of many-body interacting point particles. Four different formulations of the method are presented, which correspond to four broad categories of the application encountered in the field of plasma physics and astronomy.

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1 Introduction

Test-particle calculations have been applied to study a broad class of problems in space physics and astronomy. The underlying assumption in these applications is that, by following the evolution of particles in fields that are deemed to be good approximations of those encountered in actual systems, useful information can be inferred concerning particle kinetics. In that sense, the test-particle approach provides a first order approximation of kinetic properties of a system, given fields obtained from a macroscopic approach or from measurements. In the absence of iterations and feedback from calculated particle trajectories, the results are generally not self-consistent. The approach is nonetheless useful for understanding several aspects of particle transport and dynamics in complex systems, in which a fully consistent kinetic calculation is not practical. This approach has been applied to many problems related to particle transport and energisation in space plasmas, and it continues to be a valuable complement to large scale simulations made with fluid codes.

In the following, four types of formulations are presented that are representative of the majority of the test-particle applications encountered in the literature. These are 1) Trajectory Sampling, 2) Forward Monte Carlo, 3) Forward Liouville and 4) Backward Liouville. In Section 3, each formulation is described in detail, and illustrated with simulation results. For consistency, and in order to clearly illustrate similarities and differences between the four approaches, each method is applied to the same physical problem: that of a perpendicular plane shock in a collisionless plasma. This particular problem was chosen for its simplicity. It is nonetheless sufficient to illustrate the use of each approach, and display their similarities and differences. In this presentation, the test-particle approach is described in the context of classical mechanics of point particles with no internal degrees of freedom. These assumptions may seem somewhat restrictive. They nonetheless encompass a broad class of near Earth plasmas and astronomical applications. In cases where these assumptions need to be relaxed; for example, with ions having different ionisation stages or electron excitation levels, some of the formalism can be readily modified to accommodate for more general conditions.

The conditions of validity of the test-particle method depend on the particular formulation considered. Given the absence of iterations between the particle and current densities inferred from a test-particle calculation, and the fields used to calculate trajectories, a general condition for validity is that these fields be sufficiently close to being self-consistent. A precise assessment of this condition is difficult to make *a priori*, as a measure of self-consistency would require a fully kinetic calculation. A first assessment of consistency can be made, for example, by comparing moments of the test-particle distribution functions with corresponding quantities such as particle densities, fluxes or current densities appearing in the macroscopic models used to approximate the fields. A further assessment can, in principle be made by computing first order corrections to the fields based on the approximate plasma distribution functions obtained in the test-particle approximation. In addition to the requirement of near consistency of the fields, two of the