Commun. Comput. Phys.
doi: 10.4208/cicp.260409.120210a

Vol. 8, No. 5, pp. 1111-1138
November 2010

Fluid Structure Interaction Problems: the Necessity of a Well Posed, Stable and Accurate Formulation

Jan Nordström[1,2,3,*] and Sofia Eriksson[3]

[1] *School of Mechanical, Industrial and Aeronautical Engineering, University of the Witwatersrand, Johannesburg, WITS 2050, South Africa.*
[2] *Department of Aeronautics and Systems Integration, FOI, The Swedish Defense Research Agency, SE-164 90 Stockholm, Sweden.*
[3] *Department of Information Technology, Division of Scientific Computing, Uppsala University, SE-751 05 Uppsala, Sweden.*

Received 26 April 2009; Accepted (in revised version) 12 February 2010

Available online 23 June 2010

Abstract. We investigate problems of fluid structure interaction type and aim for a formulation that leads to a well posed problem and a stable numerical procedure. Our first objective is to investigate if the generally accepted formulations of the fluid structure interaction problem are the only possible ones. Our second objective is to derive a stable numerical coupling. To accomplish that we will use a weak coupling procedure and employ summation-by-parts operators and penalty terms. We compare the weak coupling with other common procedures. We also study the effect of high order accurate schemes. In multiple dimensions this is a formidable task and we start by investigating the simplest possible model problem available. As a flow model we use the linearized Euler equations in one dimension and as the structure model we consider a spring.

AMS subject classifications: 65M12, 65M06, 65M30, 65M20, 74F10, 35Q35

Key words: Fluid structure interaction, well posed, finite difference, high accuracy, stability.

---

1 Introduction

Fluid structure interaction (FSI) problems occur in many different areas including aerodynamics [2,4,8], acoustics [3,7,24] and medicine [9,12,14,15]. The specific FSI phenomenon called aerodynamic flutter is very dangerous and can cause accidents. It deserves attention in its own right, but it is also an example of a problem where most of the activity is due to the interaction between two separate physical problems.

*Corresponding author. *Email addresses:* Jan.Nordstrom@it.uu.se (J. Nordström), sofia.eriksson@it.uu.se (S. Eriksson)

 ©2010 Global-Science Press

Unless the coupling between these two problems is well posed and stable, either unphysical growth or damping can occur. (Some formulations try to avoid or decrease the coupling problem, see, e.g., [10]). The unphysical growth could result in expensive counter measures while the unphysical damping can provide a false sense of safety. We will investigate well-posedness and stability in detail and due to the complexity, we will start in one dimension and focus on the spatial problem.

First we investigate if the generally accepted formulation of the FSI problem is the only possible one (uniqueness) and such that it does not generate artificial energy growth. In short we will make sure that the coupling is well posed (see [11] for various concepts of well-posedness). Next, given that the problem is well posed, we will make sure that the numerical approximation is accurate and stable. We will study both the linear and non-linear problem.

The rest of the paper proceeds as follows. In Section 2 we formulate the problem mathematically. Section 3 deals with the linear problem and well-posedness while the non-linear problem is discussed in Section 4. In Section 5 we construct stable approximations for the linear semi-discrete approximation and describe the non-linear approximation. Section 6 presents the numerical experiments and conclusions are drawn in Section 7.

2 Mathematical formulation

Our model problem is schematically depicted in Fig. 1. The flow part to the left of $x_1(t)$ influence the structural part to the right and vice versa. The governing equations for the flow problem are the symmetric and linearized Euler equations in one dimension

$$U_t+AU_x=0, \quad 0\leq x\leq x_1(t), \tag{2.1}$$

where

$$A=\begin{bmatrix} \bar{u} & \bar{c}/\sqrt{\gamma} & 0 \\ \bar{c}/\sqrt{\gamma} & \bar{u} & \bar{c}\sqrt{\frac{\gamma-1}{\gamma}} \\ 0 & \bar{c}\sqrt{\frac{\gamma-1}{\gamma}} & \bar{u} \end{bmatrix}, \quad U=\begin{bmatrix} U^1 \\ U^2 \\ U^3 \end{bmatrix}=\begin{bmatrix} \bar{c}^2\rho/\sqrt{\gamma} \\ \bar{c}\bar{\rho}u \\ \bar{\rho}T/\sqrt{\gamma(\gamma-1)M_\infty^4} \end{bmatrix}. \tag{2.2}$$

In (2.2), ρ, u and T represents the perturbation from the mean value of the density ($\bar{\rho}$), velocity ($\bar{u}$) and temperature ($\bar{T}$) respectively. Furthermore, $\bar{c}$,M_∞ and γ stands for the speed of sound at reference state, the Mach number at reference state and the ratio of specific heats. The equation of state is $p=\bar{c}^2\rho/\gamma+\bar{\rho}T/(\gamma M_\infty^2)$. For more details on how to obtain (2.1), (2.2), see [1,19].

To simplify the analysis we transform equation (2.1) to a fixed domain using $\xi=x/x_1$ and $\tau=t$. We get

$$x_1(\tau)U_\tau+(A-\xi\dot{x}_1)U_\xi=0, \quad 0\leq\xi\leq 1. \tag{2.3}$$