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Numerical Simulations of Rarefied Gases in Curved Channels: Thermal Creep, Circulating Flow, and Pumping Effect

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Abstract. We present numerical simulations of a new system of micro-pump based on the thermal creep effect described by the kinetic theory of gases. This device is made of a simple smooth and curved channel with a periodic temperature distribution. Using the Boltzmann-BGK model as the governing equation for the gas flow, we develop a numerical method based on a deterministic finite volume scheme, implicit in time, with an implicit treatment of the boundary conditions. This method is comparatively less sensitive to the slow flow velocity than the usual Direct Simulation Monte Carlo method in case of long devices, and turns out to be accurate enough to compute macroscopic quantities like the pressure field in the channel. Our simulations show the ability of the device to produce a one-way flow that has a pumping effect.

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1 Introduction

In the vicinity of solid boundaries, flows of rarefied gases show a large variety of phenomena that do not exist for dense gases as described by continuous gas dynamics (like

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Stokes or Navier-Stokes equations). For instance, several effects due to a temperature field applied on a solid boundary have been observed that cannot be explained in the framework of classical gas dynamics: let us mention thermal stress slip flow, nonlinear thermal stress flow, flow induced near a heated plate edge, thermophoresis, and thermal creep flow (see Sone [35–37]). This last phenomenon was already described (as thermal transpiration) by Reynolds in 1879 [32]: in a rarefied gas contained in a pipe whose temperature has a gradient along its axis, a flow is induced in the direction of the gradient, and this flow has a pumping effect, while the device does not involve any moving part or mixing process. This was also studied by Maxwell [25], and later by Knudsen [22,23]. However, it is much more recently that a complete and accurate analysis on the basis of the Boltzmann equation has been proposed by Sone [34] (see also Ohwada, Sone and Aoki [30]). See also a continuum theory of this phenomenon proposed by Bielenberg and Brenner [8]. Recently, interest in this kind of flows is growing in connection with micro machine engineering, like Micro-Electro-Mechanical Systems (see Karniadakis, Beskok and Aluru [21] and Cercignani [13]). Indeed, the thermal creep is observed only if the gas is rarefied, that is to say when the characteristic length scale of the device containing the gas is not large with respect to its mean free path. This implies that one needs to use either very low pressure conditions, or a very small device (for instance, for the air at atmospheric pressure, the characteristic size of the device should be of the order of 0.1 microns). Different systems have recently been proposed to design pumping systems using this effect. They are often called Knudsen compressors (e.g., Pham-Van-Diep et al. [31]), since Knudsen himself in 1909 [22] described the first experimental device of this kind. The basic idea is to use cascade systems whose single unit is a pipe composed of a thin part connected to a thicker part.

In this paper, a simple device proposed in Aoki et al. [2,3] is considered: the thermal creep flow is created by applying a periodic temperature field along a simple curved channel. As opposed to previous systems, we do not use any complex connection part. Up to our knowledge, such a device had not been investigated before. Since any experimental study of such micro-systems is a difficult challenge, our aim here is to use numerical simulations to demonstrate that our device can effectively produce a one-way flow. We also want to confirm that there exists a pumping effect in the corresponding cascade system [2,3], which means that it is indeed a Knudsen compressor. However, large numerical simulations of rarefied gas problems are still a delicate issue, since this always implies using a large number of degrees of freedom. Indeed, even for two-dimensional plane flows, the distribution function of the particle velocities of the gas has six independent variables.

For that purpose, we propose a fast deterministic numerical method to accurately simulate rarefied gas flows. Actually, the most used numerical method for rarefied gases is the direct simulation Monte-Carlo method (DSMC) proposed by Bird [9]. It is a very robust and efficient method, now very well understood, in which complex physics can be included. However it remains that this method is intrinsically an unsteady method in which the numerical time scale must be smaller than the physical time scale to compute