

# A Node-Centered Artificial Viscosity Method for Two-Dimensional Lagrangian Hydrodynamics Calculations on a Staggered Grid

A. Burbeau-Augoula\*

CEA, DIF, F-91297 Arpajon, France.

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**Abstract.** This work deals with the simulation of two-dimensional Lagrangian hydrodynamics problems. Our objective is the development of an artificial viscosity that is to be used in conjunction with a staggered placement of variables: thermodynamics variables are centered within cells and position and fluid velocity at vertices. In [J. Comput. Phys., 228 (2009), 2391-2425], Maire develops a high-order cell-centered scheme for solving the gas dynamics equations. The numerical results show the accuracy and the robustness of the method, and the fact that very few Hourglass-type deformations are present. Our objective is to establish the link between the scheme of Maire and the introduction of artificial viscosity in a Lagrangian code based on a staggered grid. Our idea is to add an extra degree of freedom to the numerical scheme, which is an approximation of the fluid velocity within cells. Doing that, we can locally come down to a cell-centered approximation and define the Riemann problem associated to discrete variable discontinuities in a very natural way. This results in a node-centered artificial viscosity formulation. Numerical experiments show the robustness and the accuracy of the method, which is very easy to implement.

**AMS subject classifications:** 65M08, 76E06, 65Z05, 80M12

**Key words:** Hydrodynamics, staggered discretization, artificial viscosity, Lagrangian simulation, shock waves, finite volume scheme.

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## 1 Introduction

In the context of multimaterial flows modeling, calculations are traditionally carried out using Lagrangian numerical methods which are accurate and well tested for tracking material properties. In this paper, we are interested in Lagrangian numerical based codes for

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\*Corresponding author. *Email address:* anne.burbeau-augoula@cea.fr (A. Burbeau-Augoula)

the simulation of two dimensional hydrodynamics problems. A possible method to treat these problems is to consider a staggered localization of variables: thermodynamics variables are centered within cells and position and fluid velocity are at vertices. This allows the mesh to be trivially moved with the fluid velocity. Basic principles and difficulties in the discretization of the equations of fluid dynamics written in Lagrangian form can be found in [20]. More recently, Caramana et al. [4] showed how to construct compatible hydrodynamics algorithms. The problem of the elimination of Hourglass-type motions and artificial grid distortions is investigated in [21]. In the presence of solution discontinuities, a special treatment is necessary to model shock waves. It is usual to introduce an artificial viscosity term to smear out numerical shock profiles over a number of zones to reduce post-shock oscillations. The pioneering method is due to Von Neumann and Richtmyer and concerns one-dimensional flows [1]. They introduce an explicit artificial viscosity term to smear a shock discontinuity in space without affecting the Hugoniot conditions across the shock. The viscous stress is represented by a scalar pressure of the form

$$q = -\rho |\Delta u| \Delta u.$$

This term is quadratic in  $\Delta u$ , where  $\Delta u$  is the velocity jump across the element. The work done by the viscosity is identified with the thermodynamic irreversibility of the shock. Von Neumann and Richtmyer are at the origin of all the progress that occurred on the design of artificial viscosity afterwards. Landshoff notices in [15] that with a quadratic viscosity formulation, small oscillations still occur after the shock. He proposes a linear combination of a linear and a quadratic viscosities

$$q = C_1 \rho (\Delta u)^2 + C_2 a \rho |\Delta u|,$$

where  $a$  is the local sound speed and  $C_1$  and  $C_2$  are non-dimensional constants. In [8], Wilkins recalls that Kurapatenko has established another form of the viscosity term from the pressure jump across a shock in an ideal gas. By considering the limit of Kurapatenko's solution when the velocity jump tends to zero, a linear viscosity is obtained. The formulation reduces to a quadratic term as  $\Delta u$  becomes large. This provides a supplementary justification of the formulation proposed by Landshoff [15].

The generalization of the method for multidimensional flows raises many difficulties. The first one is concerned with the determination of the tensor character of the viscosity due to the velocity gradient. Then, one has to choose an approximation of the shock direction and a length scale. This last one has a non negligible effect on the computation robustness when quadrilateral cells have very different sizes for adjacent edges (typically large aspect ratio), which occurs very frequently in Lagrangian simulations. The simplest extension of the viscosity from one to two dimensions is to consider a viscosity as a pressure term. We have to compute a velocity jump across the shock in the element. Numerical results are strongly dependent on the approximation of the shock direction. The work associated to the viscosity term is treated as a pressure (the viscosity has the same effect in all the directions), which results in shock overheating. In [2], Schulz generalizes the scalar artificial viscosity into a tensor artificial viscosity in 2D. He develops a