Commun. Comput. Phys.
doi: 10.4208/cicp.290610.020211a

Vol. **10**, No. 5, pp. 1315-1332
November 2011

On the Importance of the Stokes-Brinkman Equations for Computing Effective Permeability in Karst Reservoirs

Marcin Krotkiewski[1,*], Ingeborg S. Ligaarden[2], Knut-Andreas Lie[2,3] and Daniel W. Schmid[1]

[1] *Physics of Geological Processes, University of Oslo, P.O. Box 1048 Blindern, NO-0316 Oslo, Norway.*
[2] *SINTEF ICT, Department of Applied Mathematics, P.O. Box 124 Blindern, NO-0314 Oslo, Norway.*
[3] *Center of Mathematics for Applications, University of Oslo, P.O. Box 1053 Blindern, NO-0316 Oslo, Norway.*

Received 29 June 2010; Accepted (in revised version) 2 February 2011

Available online 11 August 2011

Abstract. Cavities and fractures significantly affect the flow paths in carbonate reservoirs and should be accurately accounted for in numerical models. Herein, we consider the problem of computing the effective permeability of rock samples based on high-resolution 3D CT scans containing millions of voxels. We use the Stokes-Brinkman equations in the entire domain, covering regions of free flow governed by the Stokes equations, porous Darcy flow, and transitions between them. The presence of different length scales and large (ten orders of magnitude) contrasts in permeability leads to highly ill-conditioned linear systems of equations, which are difficult to solve. To obtain a problem that is computationally tractable, we first analyze the relative importance of the Stokes and Darcy terms for a set of idealized 2D models. We find that, in terms of effective permeability, the Stokes-Brinkman equations are only applicable for a special parameter set where the effective free-flow permeability is less than four orders of magnitude different from the matrix permeability. All other cases can be accurately modeled with either the Stokes or the Darcy end-member flows, depending on if there do or do not exist percolating free-flow regions. The insights obtained are used to perform a direct computation of the effective permeability of a rock sample model with more than 8 million cells.

AMS subject classifications: 65N30, 68W10, 86-02, 86-08

Key words: Upscaling, Stokes equation, Brinkman equation, porous media.

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*Corresponding author. *Email addresses:* `marcink@fys.uio.no` (M. Krotkiewski), `Ingeborg.Ligaarden@sintef.no` (I. S. Ligaarden), `Knut-Andreas.Lie@sintef.no` (K.-A. Lie), `schmid@fys.uio.no` (D. W. Schmid)

1 Introduction

Numerical simulation of flow and transport in carbonate reservoirs is a challenging problem. Carbonate reservoirs are composed of porous material, which contains cavities and fractures on multiple scales and throughout the entire rock formation. The presence of these, often relatively large void spaces affects the flow paths in the medium and should be accurately accounted for in the numerical model.

On the microscale, flow through individual pores and pore throats typically occurs at relatively low Reynold's numbers and can hence be described by the incompressible Stokes equation. On the macroscale, one can for obvious reasons not resolve the flow through individual pores and pore throats, and the flow is instead modelled using Darcy's law (and mass conservation). This macroscale description requires a set of effective petrophysical parameters-porosity and permeability-that describe the average ability of a rock to store and transport fluids, respectively. Permeability is characteristic for a given material and scale, and can be obtained from laboratory experiments, analytical upscaling formulas, or by upscaling flow simulations of rock models on a smaller scale. Herein, we consider the problem of computing such effective permeabilities for a small-scale representative elementary volume (REV) consisting of a high-resolution 3D rock sample obtained from a CT scan. The effective permeability will then be used to populate geological models on a larger scale with correct petrophysical parameters, which may then again be upscaled to enable flow simulations on the scale of the reservoir. This is typically performed using a coarse model with low numerical resolution. In this approach, due to the nature of the upscaling, all the information about the fine-scale flow structure, like fluid-front propagation and stagnant zones (important e.g., for estimating the residual fluid saturation) is lost. Other methods, such as the multiscale approach [9], aim at solving flow equations on a coarse model, while still being able to restore the fine-scale flow structure.

The standard approach to compute effective permeabilities is to solve a single-phase flow problem with a unit pressure drop applied in each axial directions of the rock sample. If the rock sample consists of only porous material, the flow inside the REV is described using the Darcy equations, as on the macro-scale. On the other hand, if the REV also contains void spaces, in which free flow may occur, one may need a more sophisticated model. To this end, there are two main approaches. The flow can either be described by a system consisting of Darcy's law for the porous material and the Stokes equation for the voids, coupled with the Beavers-Joseph-Saffman conditions on the boundaries between the porous and void volumes, see [1–4]. Alternatively, one can use the Stokes-Brinkman equations, which give a seamless transition between the Stokes and Darcy equations, see [5, 15, 16].

Herein, we assume that the Stokes-Brinkman model is best suited to describe the fine-scale flow. Irrespective of this choice, solving the flow problem numerically is a challenging task. High-resolution models obtained from CT scans typically contain millions of voxels: the particular model we consider consists of more than eight million