A Conservative Modification to the Ghost Fluid Method for Compressible Multiphase Flows

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Abstract. A conservative modification to the ghost fluid method (GFM) is developed for compressible multiphase flows. The motivation is to eliminate or reduce the conservation error of the GFM without affecting its performance. We track the conservative variables near the material interface and use this information to modify the numerical solution for an interfacing cell when the interface has passed the cell. The modification procedure can be used on the GFM with any base schemes. In this paper we use the fifth order finite difference WENO scheme for the spatial discretization and the third order TVD Runge-Kutta method for the time discretization. The level set method is used to capture the interface. Numerical experiments show that the method is at least mass and momentum conservative and is in general comparable in numerical resolution with the original GFM.

1 Introduction

Compressible multiphase flow problems are of great interest in applications, including the study of the stability of shock-interface interaction, underwater explosion and many others. Many modern Eulerian schemes exist for single phase flows. However, when solving multiphase flows, an unmodulated conservative shock capturing scheme may easily generate nonphysical oscillations near the material interfaces due to the smeared

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density and radical change in the equation of state (EOS) across the interface. Therefore, a special treatment of the material interface is often necessary [1,2]. There are two approaches to handle this problem: the front tracking method and the front capturing method. In the front tracking method, the interface is tracked as an internal moving boundary and a non-smeared interface can be materialized [8,9]. Front capturing method is easier to implement, usually the interface is implicitly tracked by using a level set function equation [3,20,21,24] or other representative function equation.

The ghost fluid method (GFM), developed by Fedkiw et al. [6,7], is a flexible way to treat compressible two-phase flows. The GFM captures the material interface by solving the level set equation and treats the interface as a boundary that separates a real fluid on one side and its corresponding ghost fluid on the other side. Both ghost fluid and real fluid exist at the grid cells and the problem near the interface essentially becomes two single-fluid problems. With properly defined ghost fluid, numerical oscillations are generally eliminated. The GFM is simple, easy to extend to multi-dimensions and can yield a sharp interface with little smearing. It can be used for two fluids of vastly different EOS. There are subsequently developed variants of the GFM which are capable of treating more extreme situations and finding wider applications [4,5,14–17,22,28].

One major drawback of the GFM is that it is a non-conservative method. The motivation of this paper is to reduce its conservation errors. We make a modification of the algorithm to obtain at least mass and momentum conservation. The original GFM uses the ghost fluid as the numerical solution for an interfacing cell at one side of the interface when the interface has left the cell. The main idea of our method is to track the conservative variables near the interface and use them to replace the numerical solution when the interface has moved away from the cell. The present method is still a GFM since the modification does not occur at every time step and does not affect the actual performance of the GFM in the interface cells. The modification procedure can be used on the GFM with any base scheme. In this paper we use the fifth order finite difference WENO scheme [12] for the spatial discretization and the third order TVD Runge-Kutta method [26] for the time discretization.

In Section 2, we introduce the governing equation and the equation of state. The ghost fluid method is also reviewed in this section. In Section 3, our numerical method and the algorithm are presented. Numerical examples including one dimensional and two dimensional test cases are given in Section 4. Section 5 contains concluding remarks.

2   The ghost fluid method

2.1   Governing equations

We consider both the one and two dimensional Euler equations. The two dimensional version is given as