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Accuracy of the Adaptive GRP Scheme and the Simulation of 2-D Riemann Problems for Compressible Euler Equations

Ee Han[1], Jiequan Li[2,*] and Huazhong Tang[3]

[1] *IAN, Department of Mathematics, Magdeburg University, D-39106, Germany.*
[2] *School of Mathematical Science, Beijing Normal University, Beijing 100875, China.*
[3] *CAPT and LMAM, School of Mathematical Sciences, Peking University, Beijing 100871, China.*

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Abstract. The adaptive generalized Riemann problem (GRP) scheme for 2-D compressible fluid flows has been proposed in [J. Comput. Phys., 229 (2010), 1448–1466] and it displays the capability in overcoming difficulties such as the start-up error for a single shock, and the numerical instability of the almost stationary shock. In this paper, we will provide the accuracy study and particularly show the performance in simulating 2-D complex wave configurations formulated with the 2-D Riemann problems for compressible Euler equations. For this purpose, we will first review the GRP scheme briefly when combined with the adaptive moving mesh technique and consider the accuracy of the adaptive GRP scheme via the comparison with the explicit formulae of analytic solutions of planar rarefaction waves, planar shock waves, the collapse problem of a wedge-shaped dam and the spiral formation problem. Then we simulate the full set of wave configurations in the 2-D four-wave Riemann problems for compressible Euler equations [SIAM J. Math. Anal., 21 (1990), 593–630], including the interactions of strong shocks (shock reflections), vortex-vortex and shock-vortex etc. This study combines the theoretical results with the numerical simulations, and thus demonstrates what Ami Harten observed *"for computational scientists there are two kinds of truth: the truth that you prove, and the truth you see when you compute"* [J. Sci. Comput., 31 (2007), 185–193].

AMS subject classifications: 65M06, 76M12, 35L60, 65M08

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*Corresponding author. *Email addresses:* `eehan84@yahoo.com` (E. Han), `jiequan@bnu.edu.cn` (J. Li), `hztang@math.pku.edu.cn` (H. Tang)

1 Introduction

The generalized Riemann problem (GRP) method was originally devised in [1] for one dimensional (1-D) system of an unsteady and inviscid flow by way of replacing the initial data with a piecewise linear function and analytically solving a generalized Riemann problem at each cell interface so as to yield numerical fluxes. Then it was extensively applied to simulating a large variety of wave configurations in 2-D and 3-D, including gas dynamics problems and combustion problems [3, 7], 2-D compressible flows with moving boundaries [23] etc. The comprehensive description can be found in [4] and references therein. This pioneering derivation has two related versions, the Lagrangian and the Eulerian. The Eulerian version is always derived by using the Lagrangian version. The approach using the Lagrangian framework has the advantage that the contact discontinuity in each local wave pattern is always fixed with speed zero and the rarefaction waves and/or shock waves are located on either side. However, the passage from the Lagrangian version to the Eulerian is sometimes quite delicate, particularly for sonic cases and multi-dimensional applications. In order to efficiently deal with the sonic cases and apply to multi-dimensional systems, the second author and his coauthors introduced a direct Eulerian GRP scheme first for the shallow water equations [35] and for the Euler equations [5, 6], which used the main ingredient of Riemann invariants to decompose the strong coupling of nonlinear waves into a form of their simple superposition so that the rarefaction waves could be analytically resolved in a quite straightforward way. The numerical implementation is almost the same as that of the linearized Euler equations, and at each cell interface only a pair of linear algebraic equations are required to be solved. In [25] this direct Eulerian version was combined with the adaptive moving mesh method [51], which consisted of two independent parts: evolution of PDEs with the GRP scheme on an quadrangular mesh and the mesh redistribution with the Gauss-Seidel iteration method. Such an adaptivity can overcome some drawbacks of many Godunov-type schemes, such as the instability of stationary shocks and start-up errors in a single shock wave simulation. Indeed, adaptive moving mesh methods have been successfully applied in a variety of scientific and engineering areas such as fluid dynamics and solid mechanics etc., where singular or nearly singular solutions are developed dynamically in fairly localized regions. To resolve the large solution variations requires extremely fine meshes over a small portion of the physical domain, see [8] for some practical examples. Successful implementation of an adaptive strategy can effectively decrease the computational cost and increase accuracy of the numerical approximations, see e.g., [19, 21, 30, 50, 51, 55, 59]. Up to now, there have been many important progresses in adaptive moving mesh methods for partial differential equations, including grid redistribution approach based on the variational principle of Winslow [56], Brackbill [9,10], Wang and Wang [55]; moving finite element methods of Miller and Miller [44], Davis and Flaherty [20]; moving mesh PDEs methods of Russell et al. [11,12]; and moving mesh methods based on the harmonic mapping of Dvinsky [22], and Li et al. [41, 42].

In [25] we have validated the efficiency of the adaptive scheme in several aspects