

Numerical Solution of the Gyroaverage Operator for the Finite Gyroradius Guiding-Center Model

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Abstract. In this work, we are concerned with numerical approximation of the gyroaverage operators arising in plasma physics to take into account the effects of the finite Larmor radius corrections. Several methods are proposed in the space configuration and compared to the reference spectral method. We then investigate the influence of the different approximations considering the coupling with some guiding-center models available in the literature.

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1 Introduction

In strongly magnetized plasma, when collision effects are negligible, one has to deal with kinetic models since fluid models, which assume that the distribution function is close to an equilibrium, are not appropriated. However, the numerical solution of Vlasov type models is challenging since this model involves six dimensions of the phase space. Moreover, multi-scaled phenomena make the problem very difficult since numerical parameters have to solve the smallest scales. Gyrokinetic theory enables to get rid of one of these constraints since the explicit dependence on the phase angle of the Vlasov equation is removed through gyrophase averaging while gyroradius effects are retained. The so-obtained five dimensional function is coupled with the Poisson equation (or its asymptotic counterpart, the quasi-neutrality equation) which is defined on the particle coordinates. Thus solving the gyrokinetic Vlasov-Poisson system requires an operator which

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carries out from the gyrocenter to the particles coordinates. This operator is the so-called gyroaverage operator. We refer to an abundant literature around this subject (see [1,10,16] and references therein).

The present work is devoted to a numerical study of the gyroaverage operator. We intend to develop and compare different methods to deal with the numerical approximation of the gyroaverage operator.

Roughly speaking, the gyroaverage process consists in computing an average on a circle (the Larmor circle). The use of Fourier transform reduces the gyroaveraging operation by a multiplication in the Fourier space by the Bessel function. In simple geometry, this can be performed easily. However, since the Larmor radius depends also on a perpendicular velocity variable μ , the use of Fourier transform is not applicable in general geometry such as those employed in realistic tokamak equilibrium. And even in simplified circular cross-section, the Fourier approach has difficulties to deal with non-periodic boundary conditions. Several approaches have thus been developed to address these problems. Approximations of the Bessel function have been proposed such as the Padé expansion (see [7]); this approximation recovers a correct behaviour of the Bessel function for small Larmor radius and also asymptotically. Moreover, it enables to come back to the spatial configuration to take into account non-periodic boundary conditions. Other works deal with quadrature formula to evaluate the integration with respect to the gyrophase angle (see [16,17]). The so-called 4-points method is quite simple since it can be expressed into a matricial formulation. However, when the Larmor radius becomes large, the method is not very accurate since the number of quadrature points is not sufficient. Moreover, when quadrature points are different from the grid points, the authors carry out a linear interpolation from the nearby grid points of the function, which can suffer from a lack of accuracy in some cases. The method has then been extended to achieve accuracy for large Larmor radius [4,12,13]. The main improvements rely on an adaptive number of quadrature points (the number gyropoints is given by an increasing function of the gyroradius [12]), but also on a finite element formalism which enables higher order accuracy keeping the matricial formulation.

In this paper, we develop and compare methods based on the direct integration of the gyroaverage operator. First, for a fixed number of quadrature points, we compare the influence of the interpolation operator (which is of great importance when the quadrature points do not coincide with the grid points). The function is reconstructed using cubic splines polynomials to reach a good accuracy (as in [18]). However, when the number of quadrature points is fixed, it is always possible to find a Larmor radius sufficiently large so that the error becomes significant. Hence, we develop a new approach, in the same spirit as [4,12,13]; the basic point is the expansion of the function on a basis (such as polynomial basis). Computing the gyroaverage of a function then reduces to compute the gyroaverage of its basis, which are known analytically. Hence, in the same way as finite element formulation, the method can be formulated into a matricial form. The approach presents other several advantages. On the one side, the number of quadrature points is automatically determined as the intersection between the mesh and the Lar-