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Gas Flow Through Square Arrays of Circular Cylinders with Klinkenberg Effect: A Lattice Boltzmann Study

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Abstract. It is well known that, as non-continuum gas flows through microscale porous media, the gas permeability derived from Darcy law is larger than the absolute permeability, which is caused by the so-called Klinkenberg effect or slippage effect. In this paper, an effective definition of Knudsen number for gas flows through square arrays of circular cylinders and a local boundary condition for non-continuum gas flows are first proposed, and then the multi-relaxation-time lattice Boltzmann equation including discrete effects on boundary condition is used to investigate Klinkenberg effect on gas flow through circular cylinders in square arrays. Numerical results show that the celebrated Klinkenberg equation is only correct for low Knudsen number, and second-order correction to Klinkenberg equation is necessary with the increase of Knudsen number. Finally, the present numerical results are also compared to some available results, and in general an agreement between them is observed.

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Key words: Klinkenberg effect, multi-relaxation-time lattice Boltzmann equation, Knudsen number, Klinkenberg equation.

1 Introduction

Gas flow in porous media has received an increasing attention for its importance and wide applications in science and engineering [1]. The study on physics of gas flow

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through porous media is necessary to understand deeply some abnormal transport phenomena (e.g., Klinkenberg effect or slippage effect) appeared in petroleum and natural gas industries. Gas slippage phenomena is usually observed in the experiments when gas flow through porous media under low pressure, at which condition the mean free path of the gas molecules is comparable with pore throat radius. Physically speaking, this slippage effect yields a larger effective gas permeability compared to that at high pressure. In the year 1941, Klinkenberg first proposed a formula on the effective gas permeability (K) as [2]

$$K = K_\infty \left(1 + \frac{b_K}{P}\right), \tag{1.1}$$

which is also called Klinkenberg equation, K_∞ is the absolute or true permeability derived under very large pressure, $b_K = 4C_0 P\lambda / r$ is the Klinkenberg factor, λ is mean free path of gas molecules, r is the effective pore radius, C_0 is a constant and P is pore pressure. By introducing the Knudsen number (Kn) which is defined as the ratio of molecular mean free path to effective pore diameter ($D_h = 2r$), the Klinkenberg equation can be rewritten as

$$K = K_\infty (1 + 8C_0 \times Kn). \tag{1.2}$$

Since the famous experiment conducted by Klinkenberg, many experimental and theoretical studies were carried out to study Klinkenberg effect, and the main focus is to present an explicit expression on Klinkenberg factor b_K [3–12], here some expressions on b_K derived by different authors are summarized in Table 1. With the help of original Klinkenberg equation or these modified equations, some macroscopic properties of flow in porous media can be derived with extended Darcy law. Due to the complexity of porous structure, however, these macroscopic expressions (e.g., extended Darcy law) have no capacity to present detailed resolutions for flow through porous media. As an efficient mesoscopic approach, the lattice Boltzmann equation (LBE) has been a new potential tool and gained much success in simulating flow through porous media at pore scale level for its distinct implementation on boundary conditions [14–21]. On the other hand, due to the kinetic background of LBE, it is also applied to study microscale gaseous flow with non-continuum effect in recent years [22–31]. Above two distinguished characteristics of LBE may make it be very suitable to investigate non-continuum gas flow in porous media.

In the past years, although many works have shown that LBE is capable of simulating flow in porous media, most of the available LBE models are constructed under the continuum assumption, and much less attention has been paid to non-continuum gas flow in porous media. To the authors' knowledge, there is only little work on applying LBE to study non-continuum gas flow in porous media [19–21]. In these published works [19–21], two basic problems have not been solved. One is how to define an effective Knudsen number for gas flow in porous media, and another is how to propose a local boundary condition for non-continuum gas flow and include its discrete effects in LBE [29–31]. Therefore, the objectives of present work are twofold. The first is to