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Gaussian Beam Summation for Diffraction in Inhomogeneous Media Based on the Grid Based Particle Method

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Abstract. We develop an efficient numerical method to compute single slit or double slit diffraction patterns from high frequency wave in inhomogeneous media. We approximate the high frequency asymptotic solution to the Helmholtz equation using the Eulerian Gaussian beam summation proposed in [20, 21]. The emitted rays from a slit are embedded in the phase space using an *open* segment. The evolution of this open curve is accurately computed using the recently developed Grid Based Particle Method [24] which results in a very efficient computational algorithm. Following the grid based particle method we proposed in [23, 24], we represent the open curve or the open surface by meshless Lagrangian particles sampled according to an underlying fixed Eulerian mesh. The end-points of the open curve are tracked explicitly and consistently with interior particles. To construct the overall wavefield, each of these sampling particles also carry necessary quantities that are obtained by solving advection-reaction equations. Numerical experiments show that the resulting method can model diffraction patterns in inhomogeneous media accurately, even in the occurrence of caustics.

AMS subject classifications: 34E05, 78A05, 78A45

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1 Introduction

Diffraction is one of the most commonly seen phenomena in wave propagation. Whenever an incident wave encounters an obstacle, it bends itself around such a small obstacle

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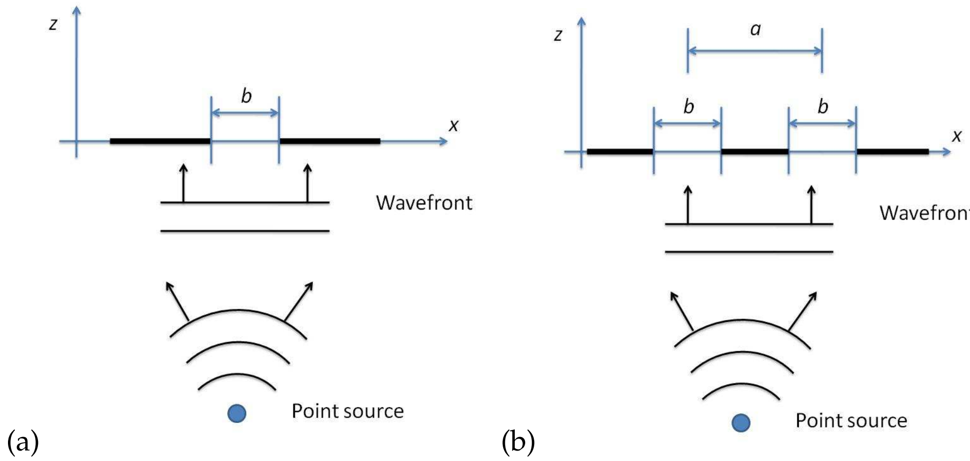

Figure 1: Setup for (a) single slit diffraction and (b) double dlit diffraction.

or it spreads itself out through small openings. In this paper, we concentrate on the second case and we will consider the single slit diffraction or the double slit diffraction. The setup of the problem is shown in Fig. 1. In both of these experiments, we for simplicity consider only that the incident wave has a plane wavefront when it hits the slit(s). This can be done by, for example, assuming that a point source is placed far away from the slit(s). We let b be the width of a slit. In double slit diffraction, we further let a be the distance between two slits. The incident wave has an incident wavelength $2\pi/\omega \ll b$.

Mathematically, such behavior cannot be predicted by solving the wave equation using the usual geometrical optics method. This high frequency asymptotic method computes wavefield only along ray trajectories which are governed by the Fremat's principle. In shadow region where no ray is reaching, the method fails to explain for the spreading of diffraction wave. There is no mechanism in usual geometrical optics to explain for the bending of ray near an obstacle.

Various methods were proposed to include the diffraction phenomenon in the high frequency asymptotic solution. One of the first attempts was the geometrical theory of diffraction (GTD) [17, 26, 32, 33]. The idea was to introduce diffracted rays in addition to the typical geometrical optics rays. These extra rays added correction to the asymptotic expansion by geometrical optics. Different types of diffracted rays were introduced to account for various kinds of scatterer surfaces [25]. However, similar to geometrical optics, typical GTD fails near caustics and special treatment has to be introduced to extend the usual GTD to compute the wavefield near these transition regions [18]. Another approach was the (ray-based) complex geometrical optics (CGO) which dealt with complex rays instead of usual real rays as in the original geometrical optics [9–11]. Diffraction of an initial beam of the Gaussian form was correctly calculated even in some inhomogeneous media [3, 4, 10, 19]. Relating to the CGO, the Gaussian beam summation method can also provide a powerful framework for constructing uniform asymptotic so-