COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **5**, No. 2-4, pp. 712-729

Commun. Comput. Phys.
February 2009

A Third-Order Upwind Compact Scheme on Curvilinear Meshes for the Incompressible Navier-Stokes Equations

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Received 30 September 2007; Accepted (in revised version) 12 April 2008

Available online 1 August 2008

Abstract. This paper presents a new version of the upwind compact finite difference scheme for solving the incompressible Navier-Stokes equations in generalized curvilinear coordinates. The artificial compressibility approach is used, which transforms the elliptic-parabolic equations into the hyperbolic-parabolic ones so that flux difference splitting can be applied. The convective terms are approximated by a third-order upwind compact scheme implemented with flux difference splitting, and the viscous terms are approximated by a fourth-order central compact scheme. The solution algorithm used is the Beam-Warming approximate factorization scheme. Numerical solutions to benchmark problems of the steady plane Couette-Poiseuille flow, the lid-driven cavity flow, and the constricting channel flow with varying geometry are presented. The computed results are found in good agreement with established analytical and numerical results. The third-order accuracy of the scheme is verified on uniform rectangular meshes.

AMS subject classifications: 76D05, 65N06

Key words: Upwind compact difference, flux difference splitting, incompressible Navier-Stokes equations, artificial compressibility, lid-driven cavity flow.

1 Introduction

The incompressible Navier-Stokes (N-S) equations are fundamental equations in fluid mechanics. Accurate numerical solution to these equations plays an important role in

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many applications. In the development of appropriate computational approaches to tackle challenging areas like direct numerical and large-eddy simulation of turbulence, high-order methods are preferable over standard second-order formulations. In the category of high-order methods, the compact difference scheme represents an attractive choice because it uses smaller stencils and gives better resolution at high wave-numbers than non-compact schemes of the same order [1–4].

The compact schemes can generally be classified into two categories: central and upwind. Central compact schemes are non-dissipative, and using central compact schemes on non-staggered meshes for convection terms might cause numerical oscillations even for flows without discontinuities. Reducing or removing such oscillations requires the use of artificial dissipation or filtering [5]. On the other hand, upwind compact schemes with dissipative properties are more stable. Fu and Ma [6,7], and among others [8–10] have developed some upwind compact schemes. Using these schemes for convective terms can provide grid-scale linkage for each variable to avoid odd-even decoupling, and in principle can prevent non-physical oscillations in smooth regions.

We note that one advantage of the upwind compact scheme by Fu and Ma [6,7] lies in that, the implicit part involves only two points rather than more points as most other upwind compact schemes do [10]. This will reduce the computational cost.

Upwind compact schemes for conservation laws require appropriate split fluxes being used. Flux vector splitting is most widely used. But fewer attempts have been made to use flux difference splitting (FDS) [11] in conjunction with upwind compact schemes. In fact, FDS is suitable for more general situations and is less dissipative than other general splitting like Lax-Friedrichs splitting. It is applicable to incompressible flows when the artificial compressibility (AC) approach is adopted. The equations for steady incompressible viscous flows with the AC formulation are (see [12])

$$\frac{\partial p}{\partial \tau}+\beta\nabla\cdot\mathbf{u}=0, \tag{1.1a}$$

$$\frac{\partial \mathbf{u}}{\partial \tau}+\mathbf{u}\cdot\nabla\mathbf{u}+\nabla p-\frac{1}{Re}\nabla^2\mathbf{u}=0, \tag{1.1b}$$

where β is the artificial compressibility parameter and τ is a pseudo-time (or iteration parameter). Since the inviscid version of the above equations are hyperbolic, upwind differences can be applied. Corresponding solution methods can be time marching ones borrowed from compressible solvers. As the solution converges, the time derivative of pressure approaches zero and the incompressibility is satisfied.

One of the disadvantages of the AC approach is the selection of the artificial compressibility parameter β. The optimum β for achieving fastest convergence is problem dependent. Through trial runs on coarse meshes, the optimum β can be found, and subsequent steady-state problems can be solved efficiently. But the AC approach is neither efficient nor accurate for time-dependent problems if sub-iteration is not converged quickly and fully. In spite of the drawbacks, numerous studies which utilized the AC approach for solving steady-state and time-dependent incompressible flow problems were