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A Multi-Mesh Adaptive Finite Element Approximation to Phase Field Models

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Abstract. In this work, we propose an efficient multi-mesh adaptive finite element method for simulating the dendritic growth in two- and three-dimensions. The governing equations used are the phase field model, where the regularity behaviors of the relevant dependent variables, namely the thermal field function and the phase field function, can be very different. To enhance the computational efficiency, we approximate these variables on different h-adaptive meshes. The coupled terms in the system are calculated based on the implementation of the multi-mesh h-adaptive algorithm proposed by Li (J. Sci. Comput., pp. 321-341, 24 (2005)). It is illustrated numerically that the multi-mesh technique is useful in solving phase field models and can save storage and the CPU time significantly.

AMS subject classifications: 65M20, 65N22, 80A22

Key words: Multi-mesh, local refinement, adaptive finite element, phase field.

1 Introduction

Dendritic growth is a main ingredient of the solidification microstructures, which has become one of the most interesting research topics in recent years. Most of the theoretical and experimental works have been devoted in understanding the mechanism of pattern selection during solidification procedure, since the microscopic properties of such

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procedure are determined by the length scale of dendrites. With the development of the theories for mathematical models of solidification, numerical simulations have become a powerful tool in investigating dendritic growth. Most theories of solidification are based on the time dependent Stefan model, which describes the evolution of thermal field around the solidification interface by the well-known heat equation with two boundary conditions: the Stefan condition [16] and the Gibbs-Thomson condition [9]. The solution of Stefan model can be approximated by that of the phase field model [4], which avoids the task of tracking the interface. The phase field model uses a phase field variable $\phi(\mathbf{r})$, which is 1 in solid phase and -1 in liquid phase. Meanwhile, the value of ϕ decreases from 1 to -1 very rapidly within a small width W near the interface; thus the level set $\phi(\mathbf{r})=0$ represents the interface implicitly.

It was originally shown by Caginalp and Chen [3] if W is much smaller than the capillary length d_0 then the phase field model converges to the sharp interface limit. With smaller value of W, more computational cost should be paid since the smallest element parameter Δx should be much smaller than W in order to fully resolve the interface. This requirement prevents us from using very small W which is close to the physical realities. Another limitation which seriously restricts the use of the phase field simulation is that the interface kinetics β should be big enough to ensure the convergence due to the Gibbs-Thomson boundary condition. But physically, it is important to simulate solidification microstructures in the limit of zero interface kinetics because most experiments performed at low undercooling are within this limit. In 1996, Wang and Sekerka [21] showed that the phase field simulations of dendritic growth are independent of computational parameters, but the results are only feasible in a range of large undercooling. Karma and Rappel [9,10] demonstrated that such limitations can be less harmful by providing the understanding in their new asymptotic analysis. It is revealed that the phase field approach can be extended to the case of arbitrary small or even zero β, and the limitation on W is not so stringent. These results made it possible to simulate more physical cases in large interface width W and low undercooling. As an important advantage of phase field model, it should be mentioned that its extension to three dimensional case is straightforward.

For solving the phase field models for dendritic solidification, the finite difference method on uniform mesh is utilized in [9, 10, 20] due to its simplicity. In the work of Provatas et al. [15], the adaptive finite element method is applied to the phase field model of dendritic growth in pure melt. The purpose of adaptive mesh refinement is to reduce computational cost so that simulations of larger scale problems become possible with currently available computational resources. In their simulations the smallest element size δx is much smaller than W; and the ratio of the system size to the smallest element size is even greater than 2^{17}. Most numerical methods will fail to work in such case, while the adaptive mesh refinement scheme can make the computation possible. The numerical results of [15] indicated the adaptive finite element method can produce quite satisfactory phase field solutions at high undercooling.

The thermal field u is one of the variables in the phase field model. It is interesting