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A Contact SPH Method with High-Order Limiters for Simulation of Inviscid Compressible Flows

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Abstract. In this paper, we study a class of contact smoothed particle hydrodynamics (SPH) by introducing Riemann solvers and using high-order limiters. In particular, a promising concept of WENO interpolation as limiter is presented in the reconstruction process. The physical values relating interactional particles used as the initial values of the Riemann problem can be reconstructed by the Taylor series expansion. The contact solvers of the Riemann problem at contact points are incorporated in SPH approximations. In order to keep the fluid density at the wall rows to be consistent with that of the inner fluid wall boundaries, several lines of dummy particles are placed outside of the solid walls, which are assigned according to the initial configuration. At last, the method is applied to compressible flows with sharp discontinuities such as the collision of two strong shocks and the interaction of two blast waves and so on. The numerical results indicate that the method is capable of handling sharp discontinuity and efficiently reducing unphysical oscillations.

AMS subject classifications: 76N15, 35L03

Key words: Meshless method, SPH method, the Riemann solution, high-order limiter, Taylor series.

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1 Introduction

In the past decade, a number of mesh-free methods such as material point method (MPM) [1], reproducing kernel particle method (RKPM) [2], smoothed particle hydrodynam-

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ics (SPH) [3], radial basis function-based differential quadrature (RBF-DQ) [4], and the method of particular solutions (MPS) [5], have become the most important research topics in computational mechanics. These methods are able to approximate an unknown function or its derivatives on a set of scattered nodes within the local support. Since the meshless methods do not require mesh for spatial discretization, they do not achieve the accuracy of the Riemann-based methods for most ideal gas problems, but they have advantages for many complex problems.

The smoothed particle hydrodynamics (SPH) method is a fully Lagrangian, meshless method which was originally devised to simulate a wide variety of problems in astrophysics involving motion of compressible fluid masses at different spatial scales. Unlike some traditional methods such as finite-difference (FD), finite volume method (FVM) and finite element method (FEM), the SPH is easy to deal with complicated flow phenomena involving arbitrary geometries. Moreover, it is simple for solving two- and three-dimensional problems. The crucial idea of the method is that a smoothing kernel is introduced to approximate functions and their spatial derivatives originating from the interactions of neighboring particles. At present the method has become a useful tool for applications in numerous domains, including free surface and interfacial flows, multi-phase, magnetohydrodynamics, high-velocity impacts, penetration, shock damage in solids and explosion phenomena.

In general, the classical SPH suffers from several perplexing problems, for example, the stability and the consistency issues. The SPH gives shock profiles that are not as sharp as those of exact Riemann solutions and that show unphysical wiggle. In order to achieve high order accuracy, a lot of reformulations of SPH for handling strong shock phenomena were reported. Campbell applied the penalty formulation to enforce the contact condition [6]. Monaghan [7] introduced an artificial viscosity term into the motion and thermal energy equations to handle shocks. Monaghan [8] devised a modified form of the dissipative terms. In this case, the SPH equations were formulated using the total energy equation rather than the thermal energy equation. Inutsuka [9], Parshikov [10, 11], Cha and Whitworth [12] proposed similar schemes where the force acting on each particle is determined by solving the Riemann problem in the vicinity of the midpoint between each pair of interacting particles. This procedure is analog to that employed in Godunov-type schemes which use a Riemann solver to calculate the flux at each cell interface. More recently, Ferrari et al. [13] devised a SPH method that relies on the use of Godunov-type schemes in Lagrangian coordinates. Above methods are the combination of standard SPH with Riemann solutions. Any pair of interacting particles is treated as the left and right states of the Riemann problem, with the changes between the two particles being taken along the line joining them. As expected, the SPH formulations based on Riemann solvers have performed well for solving sharp discontinuities of a variety of shock problems. Sigalotti [14, 15] also provided a family of formulations of SPH which do not rely on Riemann solvers but on an adaptive density kernel estimation (ADKE). The particle distribution is redefined at appropriate intervals in accordance with a previous update of smoothing length. With regard to SPH based on Riemann solvers, any pair of interacting