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A Polynomial Chaos Expansion Trust Region Method for Robust Optimization

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Abstract. Robust optimization is an approach for the design of a mechanical structure which takes into account the uncertainties of the design variables. It requires at each iteration the evaluation of some robust measures of the objective function and the constraints. In a previous work, the authors have proposed a method which efficiently generates a design of experiments with respect to the design variable uncertainties to compute the robust measures using the polynomial chaos expansion. This paper extends the proposed method to the case of the robust optimization. The generated design of experiments is used to build a surrogate model for the robust measures over a certain trust region. This leads to a trust region optimization method which only requires one evaluation of the design of experiments per iteration (single loop method). Unlike other single loop methods which are only based on a first order approximation of robust measure of the constraints and which does not handle a robust measure for the objective function, the proposed method can handle any approximation order and any choice for the robust measures. Some numerical experiments based on finite element functions are performed to show the efficiency of the method.

AMS subject classifications: 60H15, 65K05, 62K20, 62K05

Key words: Reliability based design optimization, polynomial chaos expansion, trust region method.

1 Introduction

The design of a mechanical structure is usually formulated as an optimization problem as follows:

$$\begin{aligned}&\min_x f(x)\\&\text{s.t. } g_i(x)\le 0,\quad i=1,\cdots,r,\\&\qquad x^l\le x\le x^u,\end{aligned}\tag{1.1}$$

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where $x \in \mathbb{R}^M$ is a vector of M design variables (geometry, load,$\cdots$), f is an objective function (weight, cost,$\cdots$) and g_i is a set of r constraint functions representing the mechanical requirements (deformation, buckling load,$\cdots$). Due to the uncertainties in the design variables, the f and g_i functions (the response of the mechanical system) have a deterministic component and a random component. The random component of each function is characterized by a robust measure. This robust measure consists in defining ϕ the probability density functions (pdf) of the design variable uncertainties and propagating them through the mathematical model of the mechanical system in order to characterize the random component of the output functions. In [2–4], a detailed review of the possible mathematical definitions of the robust measures is available. Let ϵ be the vector of the M design variable uncertainties. In this paper, only the case of Gaussian uncertainties is considered, but the proposed algorithm applies to any other continuous uncertainty type. For the objective function, one can define its robust measure $\mathbf{R}_f(x)$ as one of the following possibilities:

- the function itself, $\mathbf{R}_f(x) = f(x)$;
- its mean value with respect to the design variable uncertainties:

$$\mathbf{R}_f(x) = E[f|x] = \int f(x+\epsilon)\phi(\epsilon)d\epsilon;$$

- a combination of its mean value and its variance:

$$\mathbf{R}_f(x) = E[f|x] + var(f|x)$$

  with

$$var(f|x) = \int (f(x+\epsilon) - E[f|x])^2 \phi(\epsilon)d\epsilon;$$

- the probability that f is less than a certain threshold q:

$$\mathbf{R}_f(x) = Pr[f < q|x].$$

The robust measure $\mathbf{R}_{g_i}(x)$ of the constraints can be defined as:

- its statistical feasibility:

$$\mathbf{R}_{g_i}(x) = Pr[g_i \leq 0|x] \geq P_0$$

  for some confidence probability P_0;
- its feasibility robustness:

$$\mathbf{R}_{g_i}(x) = g_i(x) + \sqrt{\sum_j \Big(\frac{\partial g_i}{\partial x_j}\Big)^2} \leq 0.$$