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A Full Eulerian Fluid-Membrane Coupling Method with a Smoothed Volume-of-Fluid Approach

Satoshi Ii[1,*], Xiaobo Gong[2], Kazuyasu Sugiyama[1], Jinbiao Wu[3], Huaxiong Huang[4] and Shu Takagi[1,5]

[1] *Department of Mechanical Engineering, The University of Tokyo, 7-3-1 Hongo Bunkyo-ku, Tokyo, 113-8656, Japan.*
[2] *Department of Engineering Mechanics, NAOCE, Shanghai Jiaotong University, Shanghai 200240, China.*
[3] *LMAM & School of Mathematical Sciences, Peking University, Beijing 100871, China.*
[4] *Department of Mathematics and Statistics, York University, 4700 Keele Street, Toronto, Ontario, Canada.*
[5] *Computational Science Research Program, RIKEN, 2-1 Hirosawa Wako, Saitama, 351-0198, Japan.*

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Abstract. A novel full Eulerian fluid-elastic membrane coupling method on the fixed Cartesian coordinate mesh is proposed within the framework of the volume-of-fluid approach. The present method is based on a full Eulerian fluid-(bulk) structure coupling solver (Sugiyama *et al.*, *J. Comput. Phys.*, **230** (2011) 596–627), with the bulk structure replaced by elastic membranes. In this study, a closed membrane is considered, and it is described by a volume-of-fluid or volume-fraction information generally called VOF function. A smoothed indicator (or characteristic) function is introduced as a phase indicator which results in a smoothed VOF function. This smoothed VOF function uses a smoothed delta function, and it enables a membrane singular force to be incorporated into a mixture momentum equation. In order to deal with a membrane deformation on the Eulerian mesh, a deformation tensor is introduced and updated within a compactly supported region near the interface. Both the neo-Hookean and the Skalak models are employed in the numerical simulations. A smoothed (and less dissipative) interface capturing method is employed for the advection of the VOF function and the quantities defined on the membrane. The stability restriction due to membrane stiffness is relaxed by using a quasi-implicit approach. The present method is validated by using the spherical membrane deformation problems, and is applied to a pressure-driven flow with the biconcave membrane capsules (red blood cells).

*Corresponding author. *Email addresses:* sii@fel.t.u-tokyo.ac.jp (S. Ii), x.gong@sjtu.edu.cn (X. Gong), sugiyama@fel.t.u-tokyo.ac.jp (K. Sugiyama), jwu@math.pku.edu.cn (J. Wu), hhuang@yorku.ca (H. Huang), takagi@mech.t.u-tokyo.ac.jp (S. Takagi)

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1 Introduction

To compute the solutions of fluid-membrane interaction problems where the motion of the fluid is coupled with the motion and deformation of embedded membranes, efficient numerical methods that can be implemented easily are often required. The immersed boundary method developed by Peskin [40, 41] is one of the most successful for these types of problems. Under the framework of the immersed boundary method, the fluid equations are solved in an Eulerian frame, while the elastic membrane is tracked in a Lagrangian manner by a set of marker points. The force exerted by the membrane on the Eulerian flow field is interpolated with the smoothed (or approximate) delta function. The immersed boundary method has been applied to a wide variety of biological problems [10, 12, 14, 15]. Many refinements and extensions have been proposed over the past several decades. For example, a new version of the method was proposed in [29] that achieves the second-order accuracy for problems with smooth solutions. The front tracking method [18, 51, 52] can be applied to multi-phase flow problems including the surface tension effect with different fluid properties. Moreover, the immersed interface method [30, 31, 33] provides a recipe for developing schemes for problems with piecewise smooth solutions, by introducing the modified Taylor expansion with the interfacial jump conditions. In these schemes, the interface can be accurately represented by the Lagrangian particles, which is of particular interests when the interface has structures that are under the resolution of the fixed Eulerian mesh. On the other hand, the particle-based methods do not automatically conserve the volume or mass encompassed by the surface reconstructed from the marker particles, and the largely distorted surface meshes may lead to a numerical instability.

In order to overcome such problems, full Eulerian approaches were proposed for the fluid-structure interaction [4, 7, 9, 25, 34, 37, 38, 48, 49, 53, 54] and fluid-membrane interaction [6]. Rather than using the Lagrangian particles, field variables or functions to identify the interface are defined and updated on the Eulerian mesh. Cottet *et al.* [6] introduced the level-set function to identify the interface, in addition, the membrane stretching or variation of the surface area was obtained from the information of the level-set function. As a result, the membrane force was successfully obtained on the Eulerian mesh without using the interfacial material points. However, since the constitutive law of the membrane elasticity is limited to a model which only involves a variation of the surface area, it has not been applied yet for more general membrane models which depend on the principal strains. Recently, Sugiyama *et al.* [49] formulated a fluid-structure interaction model based on the full Eulerian framework for an incompressible fluid and