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# Study on General Governing Equations of Computational Heat Transfer and Fluid Flow

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**Abstract.** The governing equations for heat transfer and fluid flow are often formulated in a general form for the simplification of discretization and programming, which has achieved great success in thermal science and engineering. Based on the analysis of the popular general form of governing equations, we found that energy conservation cannot be guaranteed when specific heat capacity is not constant, which may lead to unreliable results. A new concept of generalized density is put forward, based on which a new general form of governing equations is proposed to guarantee energy conservation. A number of calculation examples have been employed to verify validation and feasibility.

**AMS subject classifications**: 76M12, 80A20, 68U20

**Key words**: SIMPLE algorithm, general form of governing equations, conservation, generalized variable, fluid-solid coupling.

## 1 Introduction

A general expression for governing equations is widely used in numerical simulations of heat transfer and fluid flow, in which different variables, diffusion coefficients and source terms are written in a form of a generalized variable, a generalized diffusion coefficient and a generalized source term, respectively. There are obvious advantages by utilizing

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this general form, such as unified form of discretization and programming for all the governing equations, remarkable improvement on programming efficiency, enhancement of the versatility of the program as well. The popular general governing equation is written below:

$$\frac{\partial(\rho\phi)}{\partial t}+div\,(\rho U\phi)=div\left(\Gamma_\phi{}^* grad\phi\right)+S_\phi{}^* \tag{1.1}$$

which is a classical method widely used in many textbooks [1–8], where $\phi$ is a general variable to represent variables such as $u$, $v$, $w$ and $T$, and $\Gamma_\phi{}^*$ is a generalized diffusion coefficient corresponding to the variable $\phi$. The first and second term on the left side of Eq. (1.1) are respectively the unsteady term and the convective term, while the first term on the right side is a generalized diffusion term, and the second term $S_\phi{}^*$ is a generalized source term representing the summation of those terms in the governing equations except the unsteady term, the convective term and the diffusion term. For a two-dimensional laminar fluid flow and heat transfer in a Cartesian coordinate system, the specific meanings of $\Gamma_\phi{}^*$ and $S_\phi{}^*$ are listed in Table 1 [9,10].

Table 1: Coefficient and source term of the popular general governing equations.

| Equation | $\rho$ | $\phi$ | $\Gamma_\phi{}^*$ | $S_\phi{}^*$ |
|---|---|---|---|---|
| Continuity equation | $\rho$ | 1 | 0 | 0 |
| Momentum eqn. ($x$ direction) | $\rho$ | $u$ | $\mu$ | $\rho f_x-\frac{\partial p}{\partial x}$ |
| Momentum eqn. ($y$ direction) | $\rho$ | $v$ | $\mu$ | $\rho f_y-\frac{\partial p}{\partial y}$ |
| Energy equation | $\rho$ | $T$ | $\lambda/c_p$ | $S_T/c_p$ |

In Table 1, $x$ and $y$ represent abscissa and ordinate, while $u$ and $v$ are the velocity components in the $x$- and $y$- coordinates; $f_x$ and $f_y$ are the body forces. $p$, $\rho$, $\mu$, $\lambda$ and $c_p$ respectively indicate pressure, density, dynamic viscosity, thermal conductivity and specific heat capacity. This general expression has been widely applied in numerical heat transfer to solve a large amount of practical engineering issues. However, by the theoretical analysis in this paper, it is found that energy conservation cannot be guaranteed when specific heat capacity is not a constant, and this non-conservation may lead to unreliable results, which indicates there are some limitations of the above general form of governing equations (Eq. (1.1)). In order to ensure energy conservation, this paper puts forward a new expression of general governing equations based on theoretical analysis.

## 2 Analyses of proposed form of governing equations

The conservation law of a physical variable in a finite volume would only be satisfied in numerical discretization by employing conservative governing equations [4]. According to the classical heat transfer text books [11,12], the conservative energy equation can be