

Subspace Trajectory Piecewise-Linear Model Order Reduction for Nonlinear Circuits

Xiaoda Pan¹, Hengliang Zhu^{1,*}, Fan Yang¹ and Xuan Zeng¹

¹ State Key Lab. of ASIC & System, Microelectronics Department, Fudan University, Shanghai, P.R. China.

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Abstract. Despite the efficiency of trajectory piecewise-linear (TPWL) model order reduction (MOR) for nonlinear circuits, it needs large amount of expansion points for large-scale nonlinear circuits. This will inevitably increase the model size as well as the simulation time of the resulting reduced macromodels. In this paper, subspace TPWL-MOR approach is developed for the model order reduction of nonlinear circuits. By breaking the high-dimensional state space into several subspaces with much lower dimensions, the subspace TPWL-MOR has very promising advantages of reducing the number of expansion points as well as increasing the effective region of the reduced-order model in the state space. As a result, the model size and the accuracy of the TPWL model can be greatly improved. The numerical results have shown dramatic reduction in the model size as well as the improvement in accuracy by using the subspace TPWL-MOR compared with the conventional TPWL-MOR approach.

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1 Introduction

With the continuously increasing complexity of integrated circuits, fast simulation and verification become an important part in the design flow. Model Order Reduction (MOR) is one of the most promising approaches for fast simulation of complex systems. The MOR techniques for linear time invariant systems have been well developed during the past years [1–4]. On the other hand, the problem of nonlinear system model order reduction is generally much more difficult and challenging [5–7].

*Corresponding author. *Email addresses:* 082052014@fudan.edu.cn (X. Pan), hlzhu@fudan.edu.cn (H. Zhu), yangfan@fudan.edu.cn (F. Yang), xzeng@fudan.edu.cn (X. Zeng)

Trajectory piecewise linear model order reduction (TPWL-MOR) method has been considered as the standard model order reduction approach for nonlinear circuits. In TPWL-MOR method [6], the nonlinear system is linearized around multiple expansion points on its simulated state trajectories driven by some “training inputs”. The nonlinear system is first approximated by the piecewise linear model around these expansion points, and then reduced to a smaller macromodel by model order reduction approaches. The TPWL-MOR has been widely applied in many areas including analog circuits [6,8,9], MEMS [6, 8, 10], bio-chemical system [7], thermal analysis [11], computational fluid dynamics (CFD) [12], subsurface flow simulation [13], floorplanning [14, 15], modeling oscillators [16] and analog circuits mismatch analysis [17], etc.

There have been many efforts devoted to improving the accuracy and efficiency of the TPWL-MOR approach. The trajectory piecewise linear model was extended to piecewise polynomial model in [18, 19], which improves the accuracy by using higher order bases. A wavelet-collocation based TPWL-MOR method was proposed in [20] to enhance time-domain simulation accuracy. A novel transistor level macromodel was proposed in [21] to enhance the coverage of the trajectories in high-dimensional state space for the nonlinear circuits. For finding the proper weighting function for the piecewise linear model, a time-dependent weighting function was proposed in [22] which was shown to be computationally cheaper, and a weighting scheme was proposed in [23] to enhance the stability of the reduced order model. For model order reduction of the piecewise linear model, the moment-matching approach, TBR and POD were applied in [6], [8] and [24], respectively. A localized reduction approach was developed in [25], and a nonlinear projection method was proposed in [9, 26], which can reduce the nonlinear system to much lower orders. A novel approach proposed in [27] is based on the quadratic-linear representation of nonlinear systems, which can avoid the accuracy loss and reduce the computational cost of trajectory piecewise linear approximation for a very wide range of nonlinear circuits.

One of the key steps in TPWL-MOR approaches is the interpolation of nonlinear systems in high dimensional state space. Although TPWL-MOR method avoids the exponential explosion in computational cost [6], it still needs a large amount of expansion points to cover the state trajectory of large-scale nonlinear systems. Despite many great efforts to improve the efficiency and accuracy of the TPWL model, there are very few works focusing on the direction of reducing the number of expansion points to improve the efficiency of the high-dimensional interpolation in TPWL-MOR.

The problem of high-dimensional interpolation has been studied for many decades in computational science. A novel approach that has been proposed for solving the multi-dimensional partial differential equation (PDE) is based on the ANOVA decomposition and expresses the multi-dimensional function as a combination of the functions of some subgroups of its dimensions [28]. Inspired from this work, the subspace TPWL-MOR approach is proposed in this paper to improve the efficiency and accuracy for the high-dimensional interpolation in TPWL-MOR methods. The major advantage of the subspace TPWL-MOR is that the number of expansion points for the interpolation of