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Mathematical Modelling and Numerical Simulation of Dendrite Growth Using Phase-Field Method with a Magnetic Field Effect

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Abstract. In this paper, we present a new model developed in order to analyze phenomena which arise in the solidification of binary mixtures using phase-field method, which incorporates the convection effects and the action of magnetic field. The model consists of flow, concentration, phase field and energy systems which are nonlinear evolutive and coupled systems. It represents the non-isothermal anisotropic solidification process of a binary mixture together with the motion in a melt with the applied magnetic field. To illustrate our model, numerical simulations of the influence of magnetic-field on the evolution of dendrites during the solidification of the binary mixture of Nickel-Copper (Ni-Cu) are developed. The results demonstrate that the dendritic growth under the action of magnetic-field can be simulated by using our model.

AMS subject classifications: 76W05, 35K55, 35R35, 74A50, 74N25, 65N30, 80A22

Key words: Modelling, Phase-field method, dendritic solidification, binary alloys, convection, magnetic-field, Magnetohydrodynamics, numerical simulations.

1 Introduction

In order to improve the quality and properties of mixtures, the major industrial challenges lie in the possibility to control the metal structure and its defects, that occur during the solidification process and then to achieve the desired properties in the final solidified metals.

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In recent years the so-called phase field models have become an important tool to simulate, during the solidification of pure and mixtures of materials, the formation and growth of dendrites. This approach has proved to be an emerging technology that complements experimental research. Various problems associated with phase-field formulation have been studied to treat both pure materials and binary alloys. From the theoretical or numerical simulation point view, see e.g. A. Belmiloudi et al. [5–7], D. Kessler [15], P. Laurençot [17], J. Rappaz et al. [24], S. L. Wang et al. [33] and A. A. Wheeler et al. [36]. We can note the existence of analytical solutions for this type of model, but it remains limited to very simple cases. In the case of realistic situations where the system is highly nonlinear and very complex, the numerical simulation is a necessary tool, even essential, it plays an important role in understanding and analyzing the formation of microstructures of dendrites. In this context, we can cite works of e.g., M. Grujicic et al. [10], B. Kaouil et al. [13], J. C Ramirez et al. [22, 23], M. Rappaz [25], T. Takaki et al. [29] and J. A. Warren et al. [34]. Moreover, in the last decade, the phase field method has been extended to include the effect of convection on the dendrite growth. This was motivated by the fact that during the solidification experiments it has been observed a meaningful impact of the movement in the liquid on the formation and evolution of the dendritic microstructure. For phase-field models and the simulations of dendrite growth that include the melt flow, we can cited e.g., in the case of phase-field models for the solidification of a pure metal, D. M. Anderson et al. [2] that have developed a model in which they introduced the convection using compressible Navier-Stokes equations by assuming that viscosity and density are the functions of phase-field in order to obtain the required viscosity and density variation between the two phases; R. Tonhardt et al. [31] and X. Tong al. [30] have given models by introducing convection using Navier-Stokes equations and enforcing the velocity to be zero in the solid phase. For other models which incorporate convection during the solidification, we can cite N. Al-Rawahi et al. [1], E. Bansch et al. [3]. The principle obstacle in these simulations is to compute accurately the diffusion and convection processes and to enforce the no-slip condition at the interface so that the velocity moves along-with the solid liquid interface during the solidification process.

Although significant advances in numerical simulation of microstructural evolution in the metallurgical and materials science, and therefore it is now recognized that thanks to the phase field methods, we can simulate numerically the dendritic growth in the entire domain (at the macro scale level) with actual (physically meaningful) dimensions, it still poses new and challenging problems for scientific community, because of the need to obtain approximate solutions more and more accurate and reliable. This goal can be achieved through the development of numerical analysis tools capable of reproducing fine qualitative properties and approximations of dendritic dynamics with a reasonable CPU's computation time. To reach this objective, different approaches have been recently developed, we can cite e.g. H. Wang et al. [32] in which the authors provide an *r*-adaptive moving mesh method for the quantitative phase field equations (which were provided by A. Karma et al. [14]), in both two- and three-dimensional cases. They redistribute the meshes in the physical domain by solving an optimization problem which automatically