Commun. Comput. Phys.
doi: 10.4208/cicp.030311.230911a

Vol. **12**, No. 4, pp. 1033-1050
October 2012

A New Approach to Implement Sigma Coordinate in a Numerical Model

Yiyuan Li[1], Donghai Wang[2,*] and Bin Wang[1,3]

[1] *State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China.*
[2] *State Key Laboratory of Severe Weather, Chinese Academy of Meteorological Sciences, Beijing, China.*
[3] *Ministry of Education Key Laboratory for Earth System Modeling, and Center for Earth System Science, Tsinghua University, Beijing, China.*

Received 3 March 2011; Accepted (in revised version) 23 September 2011

Communicated by Lianjie Huang

Available online 28 March 2012

Abstract. This study shows a new way to implement terrain-following σ-coordinate in a numerical model, which does not lead to the well-known "pressure gradient force (PGF)" problem. First, the causes of the PGF problem are analyzed with existing methods that are categorized into two different types based on the causes. Then, the new method that bypasses the PGF problem all together is proposed. By comparing these three methods and analyzing the expression of the scalar gradient in a curvilinear coordinate system, this study finds out that only when using the covariant scalar equations of σ-coordinate will the PGF computational form have one term in each momentum component equation, thereby avoiding the PGF problem completely. A convenient way of implementing the covariant scalar equations of σ-coordinate in a numerical atmospheric model is illustrated, which is to set corresponding parameters in the scalar equations of the Cartesian coordinate. Finally, two idealized experiments manifest that the PGF calculated with the new method is more accurate than using the classic one. This method can be used for oceanic models as well, and needs to be tested in both the atmospheric and oceanic models.

AMS subject classifications: 86-08, 86A05, 86A10

Key words: Pressure gradient force (PGF), terrain-following sigma coordinate, non-orthogonal, basis vectors, numerical modeling, computational errors.

*Corresponding author. *Email addresses:* liyiyuan@mail.iap.ac.cn (Y. Li), d.wang@hotmail.com (D. Wang), wab@lasg.iap.ac.cn (B. Wang)

1 Introduction

A terrain-following σ-coordinate is preferred in both atmospheric and oceanic models, due to its benefit of implementing boundary conditions, the concept of which was initiated by Phillips [1]. There are, however, some disadvantages associated with σ-coordinate, among which the most concerned one is the computational error associated with pressure gradient force (PGF), resulted from the computational form of PGF in σ-coordinate that has more than one term [2,3]. The PGF problem was first pointed out by Smagorinsky et al. [4].

As horizontal resolutions of numerical models increase, the PGF problem becomes more critical [5, 6]. A common view is that the PGF problem is caused by using σ-coordinate, and a number of methods have been designed to overcome this problem mostly via adjusting model parameters or constructing different algorithms for PGF terms. Corby et al. [7] was the first to design a finite difference scheme for PGF, and Gary [8] proposed to subtract reference state of density profile before calculating PGF. Qian and Zhong [9] introduced a general difference scheme for PGF focusing on the coordinate transformation near terrain. More complicated methods were subsequently proposed, such as the Jacobian method by Blumberg and Mellor [10] and the high-order schemes by McCalpin [11]. More recently, a recurrent computational methods of PGF based on hydrostatic equilibrium was proposed by Yang and Qian [12], a linear programming procedure was proposed by Sikirić et al. [13], and a perfectly balanced method for estimating PGF was suggested by Berntsen [14]. Note that all these existing methods are designed to alleviate the errors to an acceptable level, after the PGF computational form has already had more than one term. As few researches about how the problem started at the first place has been done, we try to adopt a completely new approach here, with which we obtain a set of equations in σ-coordinate that has only one term in the computational form of the PGF. In another word, we bypass the PGF problem all together.

In this study, we first explain why the PGF computational form in σ-coordinate has more than one term by means of classifying the existing methods using σ-coordinate and analyzing the expression of the scalar gradient in a curvilinear coordinate system. Based on these analyses, we elucidate that using the covariant scalar equations of σ-coordinate in a numerical model can avoid the PGF problem all together, while the simple form of boundary conditions is preserved. Finally, via two idealized experiments, we validate that the new method can calculate the PGF more precisely.

2 Analysis of the PGF problem

Many scholars have analyzed the PGF problem in σ-coordinate, and concluded that the source of the problem is the computational form of PGF being expanded into two parts. The same problem can be caused by two different ways, as we show in the later part through classifying the existing methods.