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On the Volume Conservation of the Immersed Boundary Method

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Abstract. The *immersed boundary (IB) method* is an approach to problems of fluid-structure interaction in which an elastic structure is immersed in a viscous incompressible fluid. The IB formulation of such problems uses a Lagrangian description of the structure and an Eulerian description of the fluid. It is well known that some versions of the IB method can suffer from poor volume conservation. Methods have been introduced to improve the volume-conservation properties of the IB method, but they either have been fairly specialized, or have used complex, nonstandard Eulerian finite-difference discretizations. In this paper, we use quasi-static and dynamic benchmark problems to investigate the effect of the choice of Eulerian discretization on the volume-conservation properties of a formally second-order accurate IB method. We consider both collocated and staggered-grid discretization methods. For the tests considered herein, the staggered-grid IB scheme generally yields at least a modest improvement in volume conservation when compared to cell-centered methods, and in many cases considered in this work, the spurious volume changes exhibited by the staggered-grid IB method are more than an order of magnitude smaller than those of the collocated schemes. We also compare the performance of cell-centered schemes that use either exact or approximate projection methods. We find that the volume-conservation properties of approximate projection IB methods depend strongly on the formulation of the projection method. When used with the IB method, we find that pressure-free approximate projection methods can yield extremely poor volume conservation, whereas pressure-increment approximate projection methods yield volume conservation that is nearly identical to that of a cell-centered exact projection method.

AMS subject classifications: 65M06, 65M20, 65R10, 74F10, 76D05

Key words: Immersed boundary method, fluid-structure interaction, collocated discretization, staggered-grid discretization, exact projection method, approximate projection method, volume conservation.

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1 Introduction

The *immersed boundary (IB) method* for fluid-structure interaction [1] is a mathematical formulation and numerical scheme for problems in which an elastic structure is immersed in a viscous incompressible fluid. In the IB formulation of such problems, the elasticity of the structure is described in Lagrangian form, and the momentum, velocity, and incompressibility of the coupled fluid-structure system are described in Eulerian form. In the continuous IB formulation, coupling between Lagrangian and Eulerian variables is mediated by integral equations with Dirac delta function kernels. The discrete version of the IB method employs approximations to these integral equations in which a regularized version of the delta function is used in place of the singular delta function kernels. The discretized integral equations are used to *spread* the Lagrangian forces generated by the immersed elastic structure to the Eulerian grid, and to *interpolate* the Eulerian velocity field to the nodes of the Lagrangian mesh.

It is well known that some versions of the IB method can suffer from poor volume conservation [2,3]. This lack of volume conservation manifests itself as an apparent fluid "leak" at fluid-structure interfaces, which occurs even though the Lagrangian structure moves at the local fluid velocity. Peskin and Printz [2] recognized that one cause of this lack of volume conservation is that the interpolated velocity field that determines the motion of the Lagrangian structure is not generally divergence free, even if the Eulerian velocity is divergence free with respect to the discrete divergence operator used in the numerical solution of the incompressible Navier-Stokes equations. To obtain a Lagrangian velocity field that is more nearly incompressible, Peskin and Printz constructed a modified finite-difference approximation to the Eulerian divergence operator that ensures that the interpolated velocity field is divergence free in an average sense. Their *improved volume conservation IB method* [2] uses this modified discretization to dramatically reduce the volume losses exhibited by the standard IB method. Despite the improvements in accuracy offered by this method, it does not appear to be widely used in practice. (See [4–6], however, for recent applications of the method.) A drawback of the improved volume conservation IB method that may have slowed its adoption is that it uses a complex, nonstandard finite-difference discretization of the incompressible Navier-Stokes equations. The coefficients of this modified finite-difference scheme must be derived from the form of the regularized delta function, and the resulting finite-difference operators possess broad stencils that can increase the computational expense of the method. Other, more specialized approaches to improving the volume conservation of the IB method have also been introduced, including by Newren [7] and by Stockie [8], but these methods may not be well-suited for general use.

Herein, we study the effect of the Eulerian spatial discretization on the volume conservation of a formally second-order accurate IB method in two spatial dimensions, restricting our attention to standard finite-difference schemes that are similar to discretization methods commonly used in implementations of the IB method. To minimize the differences between the discretization approaches, the Eulerian domain is taken to be periodic