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An Adaptive Mesh Refinement Strategy for Immersed Boundary/Interface Methods

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Abstract. An adaptive mesh refinement strategy is proposed in this paper for the Immersed Boundary and Immersed Interface methods for two-dimensional elliptic interface problems involving singular sources. The interface is represented by the zero level set of a Lipschitz function $\varphi(x,y)$. Our adaptive mesh refinement is done within a small tube of $|\varphi(x,y)| \le \delta$ with finer Cartesian meshes. The discrete linear system of equations is solved by a multigrid solver. The AMR methods could obtain solutions with accuracy that is similar to those on a uniform fine grid by distributing the mesh more economically, therefore, reduce the size of the linear system of the equations. Numerical examples presented show the efficiency of the grid refinement strategy.

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1 Introduction

In this paper, we develop an adaptive mesh refinement (AMR) technique for the immersed boundary (IB) method and immersed interface method (IIM) for the following elliptic interface problem:

$$\Delta u = f, \qquad (x,y) \in \Omega, \tag{1.1a}$$

$$u|_{\partial\Omega} = g, \tag{1.1b}$$

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together with the jump conditions across the interface $\Gamma: \big(X(s), Y(s)\big)$,

$$[u]_\Gamma = w(s), \qquad [u_n]_\Gamma = v(s). \tag{1.2}$$

Here, $\Omega \subset \mathbb{R}^2$ is assumed to be a rectangular domain; the interface $\Gamma \in C^2$ is a curve separating Ω into two sub-domains Ω^-, Ω^+ such that $\Omega = \Omega^- \cup \Omega^+ \cup \Gamma$.

Note that when $w(s)=0$, the above problem can be written as

$$\Delta u = f + \int_\Gamma v(s)\delta(x-X(s))\delta(y-Y(s))ds, \qquad u|_{\partial\Omega} = g. \tag{1.3}$$

This is the simplest model of Peksin's Immersed Boundary (IB) method, see for example, [7, 19–23] for the IB method, the references therein, and applications. The IB method was originally designed using Cartesian grids. To improve the accuracy of the IB method, the second order immersed interface method (IIM) [1,4,10–13] and the augmented immersed interface method (AIIM) [8, 9, 14, 16, 17] have been developed. Both IB and IIM were originally started with uniform Cartesian grids.

Advantages using uniform Cartesian grids include simplicity, robustness, and no additional cost in the grid generation for free boundary and moving interface problems. Another consideration is that many fast solvers on uniform grids can be employed.

However, uniform meshes may not be efficient or sufficient for some problems that require high resolutions in some part of the solution domain. In order to resolve the accuracy near the interface one can use refined discretization near the interface to improve the interface treatment.

Local grid refinement may be effective for interface problems since (1) often we are mainly interested in the solution near and/or on the interface; (2) the solution away from the interface is smooth enough and therefore dose not require a fine grid to resolve it; (3) often accurate gradient computation near the interface is needed. There are a few adaptive techniques developed for the IB method using Lagrangian formulation, see for example [2, 5, 6]; and for the level set method, see for example [18, 27, 28].

Many AMR techniques use information about some approximate solution to determine where to employ a local mesh refinement technique. If the location of the interface is known in advance, however, then using this a priori information to guide the AMR process may result in a more efficient method. While one AMR approach has been developed for the immersed finite element method (IFEM) in [29], no adaptive mesh refinement technique has been developed to the Immersed Interface Method using a finite difference discretization.

In this paper, we propose an adaptive mesh refinement (AMR) technique for the interface problem above for IB and IIM methods that use finite difference discretizations. Let $\varphi(x)$ be a Lipschitz continuous function whose zero level set ($\varphi=0$) is the interface. If we have a uniform Cartesian grid with mesh size h_1. We briefly outline our idea of the local refinement strategy.

- For the grid points (x_i, y_j) in the tube $|\varphi| \le \delta$, we generate a finer grid with smaller mesh size h_2 ($h_2 < h_1$).