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Inertial Frame Independent Forcing for Discrete Velocity Boltzmann Equation: Implications for Filtered Turbulence Simulation

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Abstract. We present a systematic derivation of a model based on the central moment lattice Boltzmann equation that rigorously maintains Galilean invariance of forces to simulate inertial frame independent flow fields. In this regard, the central moments, i.e. moments shifted by the local fluid velocity, of the discrete source terms of the lattice Boltzmann equation are obtained by matching those of the continuous full Boltzmann equation of various orders. This results in an exact hierarchical identity between the central moments of the source terms of a given order and the components of the central moments of the distribution functions and sources of lower orders. The corresponding source terms in velocity space are then obtained from an exact inverse transformation due to a suitable choice of orthogonal basis for moments. Furthermore, such a central moment based kinetic model is further extended by incorporating reduced compressibility effects to represent incompressible flow. Moreover, the description and simulation of fluid turbulence for full or any subset of scales or their averaged behavior should remain independent of any inertial frame of reference. Thus, based on the above formulation, a new approach in lattice Boltzmann framework to incorporate turbulence models for simulation of Galilean invariant statistical averaged or filtered turbulent fluid motion is discussed.

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Key words: Lattice Boltzmann method, central moments, Galilean invariance, turbulence, filtering.

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1 Introduction

Minimal kinetic models for the Boltzmann equation, i.e. lattice Boltzmann equation formulations, are evolving towards as alternative physically-inspired computational techniques for various fluid mechanics and other problems. Originally developed as an improved variant of the lattice gas automata [1] to eliminate statistical noise [2], the lattice Boltzmann method (LBM) has undergone a series of major refinements, in terms of its underlying physical models as well as numerical solution schemes for various applications over the last two decades [3–6]. In particular, its rigorous connection to the kinetic theory [7–9] has resulted in a number of recent developments, including models that are more physically consistent for multiphase [10, 11] and multicomponent flows [12], models for non-equilibrium phenomena beyond the Navier-Stokes-Fourier representation [13] and an asymptotic analysis approach to establish consistency of the LBM from a numerical point of view [14].

The stream-and-collide procedure of the LBM can be considered as a dramatically simplified discrete representation of the continuous Boltzmann equation. Here, the streaming step represents the advection of the distribution of particle populations along discrete directions, which are designed from symmetry considerations, between successive collisions. Much of the physical effects being modeled are represented in terms of the collision step, which also significantly influences the numerical stability of the LBM. Most of the major developments until recently were associated with the single-relaxation-time (SRT) model [15, 16] based on the BGK approximation [17], and enjoys its popularity owing, mainly, to its simplicity. However, it is prone to numerical instability. Moreover, it is inadequate in its representation of certain physical aspects, such as independently adjustable transport properties of thermal transport and viscoelastic phenomena.

These limitations have been significantly addressed in the multiple-relaxation-time (MRT) collision model [18]. This, in a sense, represents a simplified form of the relaxation LBM proposed earlier [19, 20], with an important characteristic difference in that the collision process is carried out in moment space [21] instead of in the usual velocity space. By separating the relaxation time scales of different moments, obtained by using a linear Fourier stability analysis, its numerical stability can be significantly improved [22, 23]. Furthermore, it has resulted in significant advantages over the SRT-LBM for computation of various classes of fluid flow problems, including multiphase systems [24–26], turbulent flows [27, 28] and magnetohydrodynamics [29]. It may be noted that recently a different form of MRT model based on the orthogonal Hermite polynomial projections of the distribution functions, which is independent of any underlying lattice structure, allowing representation of higher order non-equilibrium effects has been proposed [30].

The stabilization of the LBM using a single relaxation time has been addressed from a different perspective by enforcing the H-theorem locally in the collision step [31–34]. By using the attractors of the distribution function based on the minimization of a Lyapunov-type functional, non-linear stability of the LBM is achieved in this Entropic LBM. This approach has recently been significantly extended to incorporate multiple relaxation times