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Viscoelastic Immersed Boundary Methods for Zero Reynolds Number Flow

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Abstract. The immersed boundary method has been extensively used to simulate the motion of elastic structures immersed in a viscous fluid. For some applications, such as modeling biological materials, capturing internal boundary viscosity is important. We present numerical methods for simulating Kelvin-Voigt and standard linear viscoelastic structures immersed in zero Reynolds number flow. We find that the explicit time immersed boundary update is unconditionally unstable above a critical boundary to fluid viscosity ratio for a Kelvin-Voigt material. We also show there is a severe time step restriction when simulating a standard linear boundary with a small relaxation time scale using the same explicit update. A stable implicit method is presented to overcome these computation challenges.

AMS subject classifications: 65M06, 65M12, 74F10, 76D07, 76M20

Key words: Stokes flow, viscoelastic structures, fluid-structure interaction, numerical instability.

1 Background

Many challenging biological problems involve dynamic elastic structures immersed in an incompressible viscous flow. The immersed boundary method provides a way to simulate such problems. It was originally developed to simulate blood flow in the heart [23], but has been used for many biofluid applications with many different length and time scales. Some of these applications include platelet aggregation in blood clotting [10], insect flight [20], cellular biomechanics [28], cochlear dynamics [3], and many others. In each of these problems the solid structures were modeled as elastic objects.

Many biological materials are not adequately described as simply elastic materials, because they exhibit both viscous and elastic behavior [11, 26, 32]. For example, the cytoskeleton is a network of actin filaments, microtubules, and intermediate filaments that

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give animal cells their shape and ability to move [1]. On short time scales (seconds), the cytoskeleton behaves like an elastic solid, but on longer time scales (minutes) it acts like a viscoelastic fluid due to polymerization and depolymerization of actin filaments [5,18]. The cell reorganizes the cytoskeleton during motility, cell division, and mechanical interaction with its environment. The cytoskeleton has been modeled as a viscous fluid [33], viscoelastic fluid [9], viscoelastic solid [25], and elastic solid [30] depending on the context and the time scale of the cellular behavior. Elastic cytoskeletal models based on the immersed boundary formulation have been developed [4], and we are particularly interested in building upon our previous work on modeling the cytoskeleton [28]. A natural extension of these models would be to include different viscoelastic constitutive laws.

Elastic structures immersed in a viscoelastic fluid have been recently simulated [7, 29]. Viscoelasticity in structures has also been incorporated into the immersed boundary method in several ways [4, 12, 13, 16]. The cytoskeleton was modeled as a dynamic elastic network in [4], i.e. a network of elastic springs that form and break according to prescribed rules. It was shown in [4] that the material in this model behaved as a viscoelastic fluid, but it is not straightforward to extract the effective rheological properties from the local properties of the network. In [16], the authors simulate a two dimensional structure with finite mass immersed in a viscous fluid. The massive structure's motion is governed by a momentum equation that contains an internal friction proportional to the fluid velocity. A viscoelastic shell is modeled by a two dimensional elastic structure that encloses fluid in [13]. The viscous properties of the structure are inherited from the background fluid. Viscoelastic tether points where the immersed boundary force density is a function of the velocity and velocity history at boundary points were simulated in [12]. Each of these approaches included a structure viscosity in a different way. However, it is not clear how these approaches generalize to include different viscoelastic constitutive laws. A general study of viscoelastic immersed boundaries has not been done.

In this paper we investigate numerical methods to simulate a viscoelastic, massless, immersed boundary in zero Reynolds number flow. In Section 2, we present the fluid and viscoelastic constitutive equations. We consider two viscoelastic constitutive laws: the Kelvin-Voigt and standard linear models. The explicit immersed boundary algorithm is described in Section 3 with modifications for a viscoelastic boundary. Computational results for the explicit method are presented in Section 4. For the Kelvin-Voigt model, we show the explicit scheme is unconditionally unstable when the ratio of boundary to fluid viscosity is above a critical threshold. We also show the standard linear model has a severe time step restriction when the stress relaxation time scale is short. In this case, the constitutive law resembles that of the Kelvin-Voigt model. The explicit method for the Kelvin-Voigt model is analyzed in Section 5. In Section 6, we present a stable semi-implicit method to overcome the instability observed in the explicit Kelvin-Voigt method. We conclude with computational experiments and a convergence study of the semi-implicit method.