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An Optimization Method in Inverse Elastic Scattering for One-Dimensional Grating Profiles

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Abstract. Consider the inverse diffraction problem to determine a two-dimensional periodic structure from scattered elastic waves measured above the structure. We formulate the inverse problem as a least squares optimization problem, following the two-step algorithm by G. Bruckner and J. Elschner [Inverse Probl., 19 (2003), 315–329] for electromagnetic diffraction gratings. Such a method is based on the Kirsch-Kress optimization scheme and consists of two parts: a linear severely ill-posed problem and a nonlinear well-posed one. We apply this method to both smooth (C^2) and piecewise linear gratings for the Dirichlet boundary value problem of the Navier equation. Numerical reconstructions from exact and noisy data illustrate the feasibility of the method.

AMS subject classifications: 35R30, 74B05, 78A46, 35Q93

Key words: Diffraction grating, elastic waves, profile reconstruction, Tikhonov regularization, optimization method.

1 Introduction

The inverse scattering problem of recovering an unknown grating profile from the scattered field is of great importance, e.g., in quality control and design of diffractive elements with prescribed far-field patterns [8, 25]. This paper is concerned with the two-dimensional inverse elastic scattering problem for a 2π-periodic structure under the Dirichlet boundary condition, i.e., the total displacement vanishes on the scattering surface.

Existence and uniqueness results on the forward problem of elastic scattering are obtained in [2, 13, 15], while the uniqueness to the inverse problem is studied in [1] for the

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Dirichlet problem and in [14, 16] for the third and fourth kind boundary conditions. As far as we know, there does not exist any reference dealing with the inversion algorithm of determining a grating profile from scattered elastic waves for the Navier equation. The purpose of this paper is to fill this gap by extending the two-step algorithm proposed by G. Bruckner and J. Elschner in [9] to elastic scattering problems.

There is already a vast literature on the reconstruction of a perfectly conducting profile for the two-dimensional Helmholtz equation. Here we mention a conjugate gradient algorithm based on analytic continuation [21], an iterative regularization method [19], the Kirsch-Kress optimization algorithm [9–11] and the factorization method of Kirsch [5, 6]. Based on the Kirsch-Kress scheme (see [12, Chapter 5] and the references therein), a two-step algorithm for reconstructing the grating profile is proposed in [9]. The first step is to reconstruct the scattered field from near-field measurements by solving a first kind integral equation. This step is the linear severely ill-posed part and requires the Tikhonov regularization where the singular value decomposition of the integral operator is involved. The second step is to approximate the inverse solution by solving a finite dimensional least squares problem, which is non-linear but well-posed. The advantages of the two-step algorithm are the following. (i) It reduces the computational effort for the Kirsch-Kress scheme which is based on a combined cost functional that requires the determination of two unknown functions. This is mainly because the singular value decomposition of the derived first kind integral equation can be readily achieved and only the unknown grating profile function needs to be determined in the second step. (ii) One does not need to solve direct scattering problems in the process of the inversion algorithm. Note that so far the uniqueness in the inverse problem is not known for general grating profiles and we have no convergence results for the two-step algorithm. We refer to the convergence analysis in [18] for the Kirsch-Kress optimization method applied to the 2D quasiperiodic Helmholtz equation and the reconstruction of general Lipschitz grating profiles. We think that these convergence results can be extended to the elastic case.

In this paper we always assume that the incident elastic wave is an incoming pressure wave and our method can be easily extended to the case of an incident shear wave. We present numerical results for C^2-smooth and piecewise linear gratings, including the binary gratings. Note that a binary grating profile is composed of only a finite number of horizontal and vertical line segments and has many practical applications in the design of complicated grating structures. The numerical reconstruction from far-field data for several incoming pressure waves with different incident angles is also reported, which is more practical from the engineering point of view. Our numerical experiments for exact and noisy data demonstrate the efficiency and practicability of the inversion algorithm.

The paper is organized as follows. In the next section we rigorously formulate the direct and inverse elastic scattering problems for diffraction gratings. The quasiperiodic fundamental solution to the Navier equation is investigated in Section 3. In our numerical experiments we generate synthetic scattering data by solving a first kind integral equation and using the discrete Galerkin methods proposed in [7] for a smooth grating