Dynamical Motion Driven by Periodic Forcing on an Open Elastic Tube in Fluid

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Abstract. We present a three dimensional model of an open elastic tube immersed in fluid to understand valveless pumping mechanism. A fluid-tube interaction problem is simulated by the volume conserved immersed boundary method which prevents the generation of spurious velocity field near the tube and local cluster of the tube surface. In order to explain pumping phenomena without valves, average net flow is measured by changing parameter values such as pumping frequency, compression duration, and pumping amplitude. Some frequencies that make the system reach maximal or minimal net flow are selected to study case by case. We also study the effectiveness of fluid mixing using the Shannon entropy increase rate.

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Key words: Valveless pumping, volume conserved immersed boundary method, fluid mixing, average net flow.

1 Introduction

Any directional flow driven by pumping without valves in a fluid-structure mechanical system is called valveless pumping (VP). In valveless pump systems, a unidirectional net flow can be generated by applying the periodic force at an asymmetric location of the elastic tube. The closed or open valveless pump systems have been intensively studied for recent decades in computational \cite{1,3,4,8-11,14,18-21,27-29} and experimental settings \cite{4-7,12,15-17,26}. Liebau first presented the results from the initial studies on VP research. In his experiments of VP, both closed and open valveless pump systems were

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proposed to explain the VP mechanisms [15–17]. Liebau’s closed valveless pump system is composed of two different impedance materials whose elastic properties and radii of tubes are different. In particular, the studies on a closed loop of tubing that consists of two different types of homogeneous materials, soft and (almost) rigid materials, have been comprehensively performed [9, 10, 19, 21, 28]. The macro- and micro-scale experiments have been developed for biological and biomedical applications [7, 26]. There have been also proposed computational models with different space dimensions: zero-dimensional models (compartment models) [4, 9, 20], one-dimensional models [1, 3, 19, 21, 28, 29], two-dimensional [8, 10, 11, 14] and three-dimensional models [8, 18, 27]. In Liebau’s open valveless pump system, he built a model that both ends of an open elastic tube are connected to tanks as reservoirs [15–17] to generate unidirectional net flow. Recently, it has been reported that an open elastic tube made of even a single type of homogeneous material is enough to show VP phenomena [14]. In both open and closed valveless pump systems, the important features have been observed that the direction and the magnitude of a net flow are determined by the driving and geometric parameters of the valveless pump system. The pumping frequency is one of the critical parameters in determining characteristics of VP.

In this work, we have developed a three-dimensional model of an open valveless tube made of a single homogeneous material. This computational model is an extension of our two-dimensional open valveless pump model presented in [14], in which we showed the important features of VP although only one soft material was used: a unidirectional net flow is observed and its direction is very sensitive to pumping frequency. We also investigated that the direction and the magnitude of a net flow can be explained, respectively, by the sign and the amount of the work done over a cycle by a periodic force acting on the elastic tube. Even though a two-dimensional open VP model contributed to revelation of important phenomena of VP including the fluid dynamics and wall motions, there were still spatial limitations on plane. Hence, we extend our two-dimensional model to the three-dimensional model using the volume conserved immersed boundary (IB) method. This method helps to prevent the generation of spurious velocity near the tube and local cluster of the elastic tube boundary. As the earlier research on both the closed and open VP systems reported, the similar features of the systems are observed in our model. In order to demonstrate pumping phenomena, averaged net flow is measured by changing parameter values such as frequency, compression duration, and amplitude of the driving forcing. The case studies with maximal/minimal directional flows and almost zero flows are presented. We also investigate the effectiveness of fluid mixing using the Shannon entropy increase rate and observe that mixing by valveless pumping is very efficient at high Reynolds number [2]. Some selected animations for the motion of fluid dynamics inside and outside of the elastic open tube are presented in [13].

The benefits of valveless pumps are that they can generate significant flows with low power requirements and are easy to manufacture. Especially, open tubes made of a homogeneous material can be easily constructed to generate net flow and fluid mixing at specified pumping frequencies, although the quality of the material may be an issue in