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A Constrained Level Set Method for Simulating the Formation of Liquid Bridges

Dongdong He and Huaxiong Huang*

Department of Mathematics and Statistics, York University, Toronto, Ontario, Canada M3J 1P3.

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Abstract. In this paper, we investigate the dynamic process of liquid bridge formation between two parallel hydrophobic plates with hydrophilic patches, previously studied in [1]. We propose a dynamic Hele-Shaw model to take advantage of the small aspect ratio between the gap width and the plate size. A constrained level set method is applied to solve the model equations numerically, where a global constraint is imposed in the evolution [2] stage together with local constraints in the reinitialization [3] stage of level set function in order to limit numerical mass loss. In contrast to the finite element method used in [2], we use a finite difference method with a 5th order HJWENO scheme for spatial discretization. To illustrate the effectiveness of the constrained method, we have compared the results obtained by the standard level set method with those from the constrained version. Our results show that the constrained level set method produces physically reasonable results while that of the standard method is less reliable. Our numerical results also show that the dynamic nature of the flow plays an important role in the process of liquid bridge formation and criteria based on static energy minimization approach has limited applicability.

AMS subject classifications: 76T99, 65M06

Key words: Constrained level set method, Hele-Shaw, liquid bridges, mass conservation, moving interface.

1 Introduction

In recent years, rapid progresses have been made in miniature manufacturing and testing processes to take advantage of the increasing portable computing power. In [1], an experimental procedure is proposed for setting up a simple device for biomedical testing. By filling the gap between two parallel plates with a prepared solution and subsequent

*Corresponding author. *Email addresses:* ddonghe@yorku.ca (D. He), hhuang@yorku.ca (H. Huang)

formation of isolated liquid bridges between the plates, many tests can be preformed simultaneously. This was achieved by treating the two plates chemically to form circular hydrophilic patches and displacing the water-based solution with a non-immersible second fluid, e.g., silicon oil. A static model was used in [1] to predict the formation of these liquid bridges, by minimizing equilibrium surface energy. Since the formation of liquid bridges is a dynamic process, a more accurate model must go beyond the static approach.

In this paper, we adopt a dynamic Hele-Shaw model to take the advantage of the fact that the gap between the plates is small relatively to the size of the plates. And we use a level set method to numerically study the formation of liquid bridges between two patched surfaces. Furthermore, we investigate the effects of relevant physical parameters such as the viscosity ratio and capillary number, and our numerical results are consistent with those in the existing literature.

In our numerical computation, the interface between the two fluids is captured using a level set method while the Hele-Shaw equations are solved by a first order ghost fluid method [5]†. The level set method is one of the widely used approaches for problems with evolving interfaces, especially when topological changes are involved [4]. The standard level set method uses the zero level set of a smooth scalar function (usually the signed distance function) to represent the interface. The evolution of the level set function is normally governed by a transportation equation, high order methods (ENO or WENO) can be easily applied. In order to maintain the level set function as a signed distance function, a reinitialization process is needed. Although it is easy to implement, the standard level set method suffers serious mass loss. As a consequence, for a divergence-free velocity field, the computed area (volume) enclosed by the zero level set usually does not stay as a constant.

The objective of our paper is twofold. Our main purpose is to assess the importance of the dynamic aspect of the liquid bridge process that is useful for designing miniature test devices with a wide range of applications. In order to make accurate predictions, maintaining mass conservation is crucial for this class of problems and an important objective of this study is to present a method that is easy to implement and capable of handling topological changes of a moving interface. This is achieved by using a constrained level set method. While the idea of using constraints in the level set approach is not new, we have made a number of improvements in the implementations. First of all, a global constraint is imposed in the evolution stage [2] together with local constraints in the reinitialization stage [3] of level set function in order to limit numerical mass loss. Secondly, in contrast to the low order finite element method [2], we use a 5th order HJWENO scheme for spatial discretization. As a result, our version of the constrained level set method is more accurate than the existing ones. Our numerical tests based on standard test problems show that a significant improvement is achieved. Having establish the reliability of the numerical method, we carry out extensive numerical simulations of the liquid bridge

†Although high order methods such as a hybrid immersed interface level set method [6] could also be used, our focus here is to study the effect of the dynamics, and the current first order method is sufficiently accurate as long as certain precaution is taken.