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Stability Conditions for Wave Simulation in 3-D Anisotropic Media with the Pseudospectral Method

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Abstract. Simulation of elastic wave propagation has important applications in many areas such as inverse problem and geophysical exploration. In this paper, stability conditions for wave simulation in 3-D anisotropic media with the pseudospectral method are investigated. They can be expressed explicitly by elasticity constants which are easy to be applied in computations. The 3-D wave simulation for two typical anisotropic media, transversely isotropic media and orthorhombic media, are carried out. The results demonstrate some satisfactory behaviors of the pseudospectral method.

AMS subject classifications: 35L05, 65C20, 65N12, 65T99, 68U20

Key words: Wave simulation, stability conditions, 3-D, anisotropic media, pseudospectral method.

1 Introduction

Forward modeling is an important way to construct synthetic data which can be used in inverse problems, it is also a valuable way for studying wave phenomenon in complex geological structures. Various techniques for wave modeling have been developed. Such methods include the ray-tracing [8,16], finite-volume [11,41], finite-difference [1,12,18,20,25,36–39], finite-element [4,6,7,9,10,22], spectral-element [5,33] and pseudospectral methods [15,21,29,30]. In this paper, the pseudospectral or Fourier method will be used.

The ray-tracing method is based on the asymptotic solution of the eikonal equation. It has the limit of high frequency assumption. The finite-volume method adapts to unstructural grids, but constructing schemes with high-order accuracy in space is not easy. The finite-difference method is a widely used method. Its main drawback is a limitation on high-frequency resolution. Usually, ten or more grid points per wavelength are required at the Nyquist spatial frequency for the second-order explicit finite-difference method [1].

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For typical wave velocities and frequency bands in application such as in exploration seismology, it means that the grid space is the order of $3-4m$. The finite-element method is well known for its flexibility in describing problem with complex geometries. However, it is not commonly used in wave simulation. The main reason is that it requires to inverse mass matrix at each time step. In order to get an efficient scheme, the mass lumping technique is needed [4,9,40]. For low-order element such as linear Lagrange element, the mass lumping can be implemented by using the quadratic rules for numerical integration, but for high-order Lagrange element, it is not obvious and a new finite-element space is required [9]. Some comparisons between finite-element and finite-difference for solving the wave equation have already been given, for example, see [22] and [24]. The spectral-element method was first introduced by Patera [27] in computational fluid dynamics. It was first used for modelling wave propagation by Seriani et al. [33]. Like the finite-element method, the mass lumping is also used in the spectral-element method [5].

The pseudospectral method or Fourier method was introduced in early 1970s [13, 26]. Fornberg discussed the basic features of pseudospectral method and compared the method with the finite-difference method for the 2-D elastic wave equation [14]. He pointed out in the case of smoothly varying coefficients the required grid spacings in each space dimension satisfied ratios of $16:4:1$ for the pseudospectral method, fourth-order difference, and second-order finite-difference. The pseudospectral method differs from the finite-difference technique is that it uses the fast Fourier transform (FFT) to calculate spatial derivatives instead of finite-difference. The resulting derivative operators are highly accurate, and only two grid points are required to resolve a spatial wave length. The pseudospectral method can be viewed as the limit of finite-difference with infinite order of accuracy. Usually, high accuracy in spatial approximation is the primary pursuit in wave simulation. This is the main reason why we use the Fourier method in this paper.

Anisotropy is existed widely in the earth. For example, sedimentary rocks frequently possess an anisotropic structure [31]. In a completely anisotropic medium, 21 elastic constants are necessary to correctly define the medium [3]. Body symmetries reduce the number of independent elastic parameters. There are two typical and important anisotropic medium, transversely isotropic (TI) media and orthorhombic anisotropy (OA) media. Transversely isotropic media exhibits hexagonal symmetry that reduces to 5 the number of independent elastic constants, while OA media has 9 independent elastic constants.

In this paper, the pseudospectral or Fourier method is applied to simulating wave propagation in 3-D anisotropic media. Wave simulation with this method has been done for acoustic and elastic isotropic media [21,29,30], however, to my knowledge, the work of 3-D wave simulation in TI and OA media is few and its stability analysis is still a blank. I focus attentions on the stability analysis of 3-D numerical simulation. The stability conditions for OA and arbitrary anisotropic media are investigated. They can be expressed explicitly with elastic constants. Numerical computations for two typical 3-D anisotropic models, transversely isotropic media and orthorhombic anisotropy media, are implemented. The results show the corrects and effects of our algorithm and analysis.