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An Application of the Level Set Method to Underwater Acoustic Propagation

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Abstract. An algorithm for computing wavefronts, based on the high frequency approximation to the wave equation, is presented. This technique applies the level set method to underwater acoustic wavefront propagation in the time domain. The level set method allows for computation of the acoustic phase function using established numerical techniques to solve a first order transport equation to a desired order of accuracy. Traditional methods for solving the eikonal equation directly on a fixed grid limit one to only the first arrivals, so these approaches are not useful when multi-path propagation is present. Applying the level set model to the problem allows for the time domain computation of the phase function on a fixed grid, without having to restrict to first arrival times. The implementation presented has no restrictions on range dependence or direction of travel, and offers improved efficiency over solving the full wave equation which under the high frequency assumption requires a large number of grid points to resolve the highly oscillatory solutions. Boundary conditions are discussed, and an approach is suggested for producing good results in the presence of boundary reflections. An efficient method to compute the amplitude from the level set method solutions is also presented. Comparisons to analytical solutions are presented where available, and numerical results are validated by comparing results with exact solutions where available, a full wave equation solver, and with wavefronts extracted from ray tracing software.

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1 Introduction

In this work, a fixed-grid model is applied to computational, high-frequency, underwater acoustic propagation. The proposed method builds upon the foundation established by Osher, Cheng, Kang, Shim, and Tsai [1] in which a level set method for geometric optics was introduced. High frequency propagation modeling in underwater acoustics is traditionally accomplished via ray tracing. Rather than solve for the acoustic pressure directly, the geometric optics approximation to the wave equation is employed to solve for a more slowly varying phase function and a separate amplitude function. Ray tracing solves the eikonal equation for the phase using the method of characteristics. When rays (characteristics) diverge, eventually they do not cover enough physical space, and well-resolved solutions are not available on any uniform grid.

Several computational approaches in addition to ray tracing already exist which can accurately solve the equations of acoustic propagation. However, these are not appropriate methods at high frequencies where required grid sizes become large enough to overwhelm computational resources. Ray tracing is therefore the current standard for high frequency or long range propagation modeling in underwater acoustics. The level set method may provide a practical alternative to ray tracing for solving the high frequency approximation to the wave equation for certain applications in which the need for control over solution accuracy is balanced against the need for computational speed. Such applications include modeling propagation in shallow water environments where multi-path propagation leads to combinatorial expense in the tabulation of returns from numerous eigenrays (one-way source to receiver paths). The effects of source and receiver beam patterns combined with the divergence of rays from the source often lead to poor reconstruction of the pressure field, especially in the presence of variable bathymetry or surface waves.

The difficulties with the Lagrangian approach are familiar from studies of long range acoustic propagation. The ray chaos problem was discussed in [2]. The term "ray chaos" generally refers to the phenomenon whereby small perturbations in the ray shooting angle result in large variations in the resulting trajectories. When chaotic rays are present, a high degree of precision is required to specify shooting angles in order to be able to locate eigenrays. In [3], Collins and Kuperman suggested an alternative method to compute eigenrays in the presence of ray chaos, i.e., the boundary value problem perspective vice an initial value problem (e.g., shooting method) for locating eigenrays, but their method relied on direct path optimization and did not allow for bottom or surface reflections. Godin [4] examined the behavior of rays versus that of wavefronts under weak sound speed fluctuations and showed that wavefronts are much more stable than rays, in the sense that the significant ray perturbations tend to occur along the wavefronts rather than across them. These results suggest that a propagation model based on acoustic wavefronts would be a useful tool for the underwater acoustics community.

The level set method is a wavefront-based model. By solving on a fixed grid in the phase space and evolving entire wavefronts in time, the eigenray (boundary-value) prob-