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Computational Aspects of Multiscale Simulation with the Lumped Particle Framework

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Abstract. First introduced in [2], the lumped particle framework is a flexible and numerically efficient framework for the modelling of particle transport in fluid flow. In this paper, the framework is expanded to simulate multicomponent particle-laden fluid flow. This is accomplished by introducing simulation protocols to model particles over a wide range of length and time scales. Consequently, we present a time ordering scheme and an approximate approach for accelerating the computation of evolution of different particle constituents with large differences in physical scales. We apply the extended framework on the temporal evolution of three particle constituents in sand-laden flow, and horizontal release of spherical particles. Furthermore, we evaluate the numerical error of the lumped particle model. In this context, we discuss the Velocity-Verlet numerical scheme, and show how to apply this to solving Newton's equations within the framework. We show that the increased accuracy of the Velocity-Verlet scheme is not lost when applied to the lumped particle framework.

AMS subject classifications: 65L12, 65L70, 68W25, 86-08, 34E13, 34E99

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1 Introduction

A wide range of scientific and engineering challenges involve physics on multiple time and length scales. This is particularly the case with the modelling of particle-laden fluids.

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These flows often involve physical processes on widely different length and time scales. One specific challenge lies in the fact that particle size rarely is uniform, often varying many orders of magnitude. Consequently, multiscale methodologies must be developed to be able to resolve the physics of these flows. From a computational point of view, new numerical strategies are required to deal with this class of problems [1, 5]. This paper attempts to answer some of the questions that arise in multicomponent modelling of sand-laden fluid flow. The analysis of these issues will be done within the lumped particle model. Specifically, we will focus on the computational challenges of multiscale modelling within this framework.

One of the challenges in modelling sand-laden flows is that the physical processes involved interact over many different length and time scales. For instance, the dynamics of the fluid and the suspended particles occur on time scales which may differ in many orders of magnitude. Moreover, the variable size of the suspensions adds another layer of complexity. The particle sizes range from smallest clay of micrometer in diameter to the much larger millimeter-sized gravel. Recall Fig. 1 that shows the relative size differences in these particles. The effect of a constant external force on the different particle types will typically lead to different resultant outcomes. Moreover, the fluid phase evolves in its own temporal scale which often differs from that of the particles. We will in this section describe one way of simulating such multiscale physics with the lumped particle model.

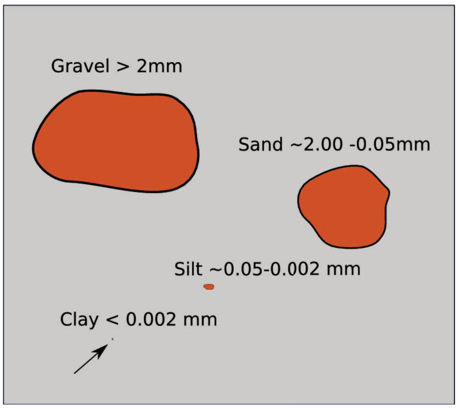

Figure 1: Typical sand particles suspended in fluids.

The lumped particle model is a flexible and numerically efficient framework for the modelling of particle transport in fluid flow, which takes into account fundamental features of particle flow, including advection, diffusion and dispersion of the particles. This framework reproduces particle flow properties inherent in both continuum and discrete approaches, and correctly reproduces advection and diffusion phenomena as special cases [2].

This paper will study computational aspects of multiscale modelling with the lumped particle framework. Specifically, we will study how an implementation of a sand-laden