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Numerical Simulation of Compressible Vortical Flows Using a Conservative Unstructured-Grid Adaptive Scheme†

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Abstract. A two-dimensional numerical scheme for the compressible Euler equations is presented and applied here to the simulation of exemplary compressible vortical flows. The proposed approach allows to perform computations on unstructured moving grids with adaptation, which is required to capture complex features of the flow-field. Grid adaptation is driven by suitable error indicators based on the Mach number and by element-quality constraints as well. At the new time level, the computational grid is obtained by a suitable combination of grid smoothing, edge-swapping, grid refinement and de-refinement. The grid modifications—including topology modification due to edge-swapping or the insertion/deletion of a new grid node—are interpreted at the flow solver level as continuous (in time) deformations of suitably-defined node-centered finite volumes. The solution over the new grid is obtained without explicitly resorting to interpolation techniques, since the definition of suitable interface velocities allows one to determine the new solution by simple integration of the Arbitrary Lagrangian-Eulerian formulation of the flow equations. Numerical simulations of the steady oblique-shock problem, of the steady transonic flow and of the start-up unsteady flow around the NACA 0012 airfoil are presented to assess the scheme capabilities to describe these flows accurately.

AMS subject classifications: 52B10, 65D18

Key words: Moving grid method, conservative interpolation.

1 Introduction

The accurate prediction of the trailing vortexes from airplane wings and helicopter blades is of paramount importance for the determination of the aerodynamic characteristics of the aircraft [18]. For example, in air traffic control, the persistence over the airport of

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start-up vortexes originating from lift strongly limits the availability of airstrips. In helicopters aerodynamics, the accurate evaluation of the unsteady dynamics of blade tip vortexes is relevant to the prediction of the aerodynamic loads of the blade and of the so called blade vortex interaction phenomenon, whose occurrence prevents the widespread use of helicopters in urban environment [12]. In nature and in man-made machinery, countless examples of fluid flows can be found for which the accurate evaluation of vortex dynamics is fundamental, including the lift due to wing flapping in insect and bird flight, thrust generated by tail flapping in fishes, unsteady wind loads caused by alternate vortex separation on slender structures such as bridges or towers or the mutual influence of wind turbine in wind farms.

From a numerical point of view, this kind of flows present peculiarities that make them difficult or impossible to simulate accurately. Indeed, different geometrical scales coexist in the flow field which strongly influence each other. For example, the wake dynamics past the separation point in separated or recirculating flows determines in a coupled manner the position of the separation line itself. Moreover, the discrete representation of slip lines, which requires high spatial accuracy, cannot be easily accomplished by the use of high-order spatial discretization or so-called p-refinement. In compressible flows, further difficulties are encountered due to possible occurrence of nonlinear wave-fields including shocks. Indeed, stabilization techniques used to capture shock wave fronts without spurious oscillations, such as for example Total Variation Diminishing schemes [19, 21], usually produce inaccurate results if applied to linearly-degenerate waves such as contact discontinuities or slip lines, thus making it necessary to locally adapt the computational grid close to discontinuities to reduce the amount of numerical viscosity.

A two-dimensional adaptive-grid numerical scheme for the compressible Euler equations is applied here to the evaluation of the start-up vortex and vortical wake from two-dimensional airfoils [8, 9]. Grid adaptation is driven by suitable error indicators based on the Mach number and by element-quality constraints as well. The error indicator is computed by means of a node-pair finite element approach [17]. At the new time level, the computational grid is obtained by a suitable combination of grid smoothing, edge-swapping, grid refinement and coarsening. These modification to the grid are interpreted at the flow solver level as continuous (in time) deformations of suitably-defined node-centered finite volumes. Therefore, the solution over the new grid is obtained without explicitly resorting to interpolation techniques, since the definition of suitable interface velocities allows to determine the new solution by simple integration of the Arbitrary Lagrangian-Eulerian formulation of the flow equations.

The present paper is structured as follows. First, the grid alteration strategy is presented and the different error indicators are discussed. The edge-based ALE solver is briefly recalled and the solution technique for adaptive moving grid is sketched. Numerical results for the oblique-shock problem and for the NACA 0012 airfoil in steady and unsteady flows are then reported and discussed. The paper ends with some final remarks and considerations.