Commun. Comput. Phys.
doi: 10.4208/cicp.010111.230911a

Vol. **12**, No. 4, pp. 1006-1032
October 2012

A Strong Stability-Preserving Predictor-Corrector Method for the Simulation of Elastic Wave Propagation in Anisotropic Media

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Received 1 January 2011; Accepted (in revised version) 23 September 2011

Communicated by Lianjie Huang

Available online 28 March 2012

Abstract. In this paper, we propose a strong stability-preserving predictor-corrector (SSPC) method based on an implicit Runge-Kutta method to solve the acoustic- and elastic-wave equations. We first transform the wave equations into a system of ordinary differential equations (ODEs) and apply the local extrapolation method to discretize the spatial high-order derivatives, resulting in a system of semi-discrete ODEs. Then we use the SSPC method based on an implicit Runge-Kutta method to solve the semi-discrete ODEs and introduce a weighting parameter into the SSPC method. On top of such a structure, we develop a robust numerical algorithm to effectively suppress the numerical dispersion, which is usually caused by the discretization of wave equations when coarse grids are used or geological models have large velocity contrasts between adjacent layers. Meanwhile, we investigate the performance of the SSPC method including numerical errors and convergence rate, numerical dispersion, and stability criteria with different choices of the weighting parameter to solve 1-D and 2-D acoustic- and elastic-wave equations. When the SSPC is applied to seismic simulations, the computational efficiency is also investigated by comparing the SSPC, the fourth-order Lax-Wendroff correction (LWC) method, and the staggered-grid (SG) finite difference method. Comparisons of synthetic waveforms computed by the SSPC and analytic solutions for acoustic and elastic models are given to illustrate the accuracy and the validity of the SSPC method. Furthermore, several numerical experiments are conducted for the geological models including a 2-D homogeneous transversely isotropic (TI) medium, a two-layer elastic model, and the 2-D SEG/EAGE salt model. The results show that the SSPC can be used as a practical tool for large-scale seismic simulation because of its effectiveness in suppressing numerical dispersion even in the situations such as coarse grids, strong interfaces, or high frequencies.

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AMS subject classifications: 65M06, 65M12, 86-08, 86A15

Key words: SSPC method, seismic wavefield modeling, anisotropy, numerical dispersion, shear-wave splitting.

1 Introduction

The development of good numerical methods is always important for both forward modeling and inverse problems (e.g. reverse time migration in exploration geophysics). Many numerical methods have been developed and widely applied in seismology. The Finite difference (FD) method such as compact FD methods (e.g. [5, 9, 16, 24]) and staggered-grid FD methods [8, 26] are widely used because of their fast speed and low computational memory for the same grid-point number, but they suffer from serious numerical dispersion when too coarse grids are chosen or too few samples per wavelength are used. The finite-element method (FEM) [4, 21, 25, 27] is a variational method which can flexibly handle variable boundary conditions with complex topography, but it requires solving large-scale linear algebraic equations at each step of time advancing which leads to large amounts of direct-access memory and computational time. The spectral method has a good property of exponential convergence rate by introducing a global basis. However, it requires the fast Fourier transform (FFT) which is time consuming, and it is also affected by numerical dispersion in time [13, 14, 29]. The spectral element method (SEM) [13, 14, 17, 18] solves the wave equations in a framework of variational method and introduces a high accuracy of spectral techniques. The SEM inherits the advantages of both the FEM and spectral method for its exponential convergence rate and high accuracy. Although the SEM uses some diagonalizing skills to reduce the bandwidth of its mass matrix, it still needs to solve a system of linear algebraic equations at each time step, resulting in costly calculation time.

Many researchers have concerned with reducing the numerical dispersion in wavefield modeling especially when coarse grids are used or models have large velocity contrasts [2,7,20,28,33,34]. Roughly speaking, numerical dispersion is actually an unphysical waves caused by the discretization of the wave equations in which the numerical wave-velocity depends on spatial and time increments, and frequency. This unphysical wave oscillation affects our recognition of seismic-wave propagation. In some extend, high-order finite difference methods can reduce the numerical dispersion, but they usually involve more grid points in a spatial direction, resulting in both the difficulty of artificial boundary treatments and reducing the efficiency for parallel calculations. The flux-corrected transport (FCT) technique was proposed to eliminate the numerical dispersion, but it cannot fully recover the lost resolution when too coarse grids are used [7,28]. To effectively suppress the numerical dispersion to solve acoustic- and elastic-wave equations, the so-called "nearly analytic discrete method (NADM)" and its improved versions have been developed in recent years by Yang *et al.* (e.g. [3, 29–31]). Owing to the validity of