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A High Order Spectral Volume Formulation for Solving Equations Containing Higher Spatial Derivative Terms II: Improving the Third Derivative Spatial Discretization Using the LDG2 Method

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Abstract. In this paper, the second in a series, we improve the discretization of the higher spatial derivative terms in a spectral volume (SV) context. The motivation for the above comes from [J. Sci. Comput., 46(2), 314–328], wherein the authors developed a variant of the LDG (Local Discontinuous Galerkin) flux discretization method. This variant (aptly named LDG2), not only displayed higher accuracy than the LDG approach, but also vastly reduced its unsymmetrical nature. In this paper, we adapt the LDG2 formulation for discretizing third derivative terms. A linear Fourier analysis was performed to compare the dispersion and the dissipation properties of the LDG2 and the LDG formulations. The results of the analysis showed that the LDG2 scheme (i) is stable for 2^{nd} and 3^{rd} orders and (ii) generates smaller dissipation and dispersion errors than the LDG formulation for all the orders. The 4^{th} order LDG2 scheme is however mildly unstable: as the real component of the principal eigen value briefly becomes positive. In order to circumvent the above, a weighted average of the LDG and the LDG2 fluxes was used as the final numerical flux. Even a weight of 1.5% for the LDG (i.e., 98.5% for the LDG2) was sufficient to make the scheme stable. This weighted scheme is still predominantly LDG2 and hence generated smaller dissipation and dispersion errors than the LDG formulation. Numerical experiments are performed to validate the analysis. In general, the numerical results are very promising and indicate that the approach has a great potential for higher dimension Korteweg-de Vries (KdV) type problems.

AMS subject classifications: 65

Key words: Spectral volume, LDG2, LDG, higher spatial derivative terms, KDV, Fourier analysis.

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1 Introduction

We continue with the development of the spectral volume (SV) method for solving equations containing higher spatial derivative terms, following the first paper in the series [17], wherein a LDG flux discretization method was employed for handling equations containing third derivative terms. The ultimate goal of this research study is to have a spectral volume formulation for equations containing higher spatial derivative terms, with the following attributes: (a) high order accurate; (b) easily applicable to multi dimensional problems; (c) geometrically flexible; (d) easily hook up with an implicit solver and algebraic, geometric and polynomial multigrid preconditioners and (e) easily extendable (eventually) for even higher (fourth or more) spatial derivative terms.

The spectral volume method was originally formulated Wang et al. [25, 31–35] and further developed by Kannan et al. [12–22] for conservation laws on unstructured grids. The spectral volume method can be viewed as an extension of the Godunov method to higher order by adding more degrees-of-freedom (DOFs) in the form of sub cells in each cell (simplex). The simplex is referred to as a spectral volume (SV) and the subcells are referred to as control volumes (CV). All the SVs are partitioned in a geometrically similar manner in a simplex, and thus a single reconstruction is obtained. The DOFs are then updated to high-order accuracy using the usual Godunov method.

The SV method was successfully implemented for 2D Euler [34] and 3D Maxwell equations [25]. The quadrature free formulation was implemented by Harris et al. [9]. A *h-p* adaptation was also carried out in 2D [10]. Recently Sun et al. [29] implemented the SV method for the Navier Stokes equations using the LDG [7] approach to discretize the viscous fluxes. Kannan and Wang [14, 22] conducted some Fourier analysis for a variety of viscous flux formulations. Kannan implemented the SV method for the Navier Stokes equations using the LDG2 (which is an improvised variant of the LDG approach) [15] and DDG approaches [16]. Even more recently, Kannan extended the SV method to solve the moment models in semiconductor device simulations [12, 13]. A new high order boundary condition was developed in the SV context for inviscid flows by Kannan [18]. A SV formulation for the line contact Elastohydrodynamic Lubrication problem was developed by Kannan [19].

In this paper, we adapt the LDG2 formulation for solving equations containing third spatial derivative terms in a SV context. The LDG2 formulation was recently proposed by Kannan and Wang [15], as an improvement to the traditional LDG formulation. The LDG2 formulation is more symmetrical and displays higher accuracy than the LDG formulation. Fourier analysis was performed on the LDG and the new variant (LDG2) and these yielded some interesting results on accuracy and stability of the formulation. Numerical tests were performed to confirm the above.

The paper is organized as follows. In the next section, we review the basics of the SV method. The LDG formulation for high order spatial derivatives is presented in Section 3. A detailed linear analysis is performed for the LDG formulation in Section 4. Section 5 presents with the different test cases conducted in this study. Finally conclusions from