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Second-Order Two-Scale Analysis Method for the Heat Conductive Problem with Radiation Boundary Condition in Periodical Porous Domain

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Abstract. In this paper a second-order two-scale (SOTS) analysis method is developed for a static heat conductive problem in a periodical porous domain with radiation boundary condition on the surfaces of cavities. By using asymptotic expansion for the temperature field and a proper regularity assumption on the macroscopic scale, the cell problem, effective material coefficients, homogenization problem, first-order correctors and second-order correctors are obtained successively. The characteristics of the asymptotic model is the coupling of the cell problems with the homogenization temperature field due to the nonlinearity and nonlocality of the radiation boundary condition. The error estimation is also obtained for the original solution and the SOTS's approximation solution. Finally the corresponding finite element algorithms are developed and a simple numerical example is presented.

AMS subject classifications: 35B27, 74Q05, 83C30, 34E13

Key words: Periodic structure, porous material, radiation boundary condition, second-order two-scale method.

1 Introduction

Porous materials have many elegant qualities, such as low relative density, heat insulation etc, and have been widely used in high technology engineering. As the materials often have periodic configurations and the coefficients change rapidly in small cells, it is needed to develop new effective numerical methods for predicting the physical and mechanical performance.

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Based on the homogenization method proposed [2–6], the Second-Order Two-Scale Analysis Method (SOTS) is introduced by Cui and Cao [15–18] to predict the physical and mechanical behavior of the materials. By second order corrector, the microscopic fluctuation of physical and mechanical behaviors inside the material can be captured more accurately. In those methods, the original problem can be approximately solved by solving a homogenized problem in original domain without holes and a series of cell problems only in one normalized cell.

As we all known that some extreme conditions are often encountered in the modern engineering. For example, the spacecraft's flying out or reentry into the atmosphere, its surface will bear strong aerodynamic force and heat. Under such conditions, the heat radiation should not be omitted. Because of its nonlinearity, it is difficulty to solve this kind of problems.

In the study of heat transfer model, there are few results concerning the heat radiation. Tiihonen [9] discussed the radiation on non-convex surfaces, and proved the existence and uniqueness of the stationary conduction radiation problem, Bachvalov [7] studied an averaging method on the heat transfer process inside periodic media with radiation and gave the asymptotic expansion of the temperature. Allaire and Ganaoui [8] studied the homogenization method of heat transfer problem with radiation on the surface of the cavities by a scaling hypothesis, and gave the homogenized solutions and first-order two-scale approximate solution, but higher order correctors are not presented.

It should be noted that if substituting the first-order two-scale solution into original equation, one can find that the residual is $\mathcal{O}(1)$ even though H^1 norm of its error is $\mathcal{O}(\varepsilon^{1/2})$. In practical engineering computation, however, ε is a constant less than the structural size L and does not tend to zero. So the local error $\mathcal{O}(1)$ is not accepted for engineer who wants to capture the local behavior of the solution. In this paper, the second-order two-scale approximation solution is discussed even though its convergence order is $\mathcal{O}(\varepsilon^{1/2})$ yet.

The remainder of this paper is organized as follows: The heat transfer model with radiation boundary condition is discussed in Section 2. The second-order two-scale asymptotic analysis for the model is presented in Section 3. The error estimation on the asymptotic solution is analyzed in Section 4. The second-order two-scale algorithm and a simple numerical example are shown in Section 5, followed by conclusions.

Throughout this paper, C (with or without subscripts) denotes a generic positive constant with possibly different values in different contexts. By $\mathcal{O}(\varepsilon^k)$, $k\in\mathbb{N}$, we denote that there exists a constant C independent of ε and $|\mathcal{O}(\varepsilon^k)|\leq C\varepsilon^k$. Also we use convention of summation on repeated indices.

2 Heat transfer model with radiation boundary condition

2.1 Periodical porous materials and Radiative boundary condition

The materials occupy a periodical porous domain in two dimension, let ω be invariant under the shifts by any $z=(z_1,\cdots,z_n)\in\mathbb{Z}^n$.