

Weighted Interior Penalty Method with Semi-Implicit Integration Factor Method for Non-Equilibrium Radiation Diffusion Equation

Rongpei Zhang^{1,3,*}, Xijun Yu², Jiang Zhu³, Abimael F. D. Loula³ and Xia Cui²

¹ School of Sciences, Liaoning ShiHua University, Fushun 113001, China.

² Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, China.

³ Laboratório Nacional de Computação Científica, MCTI, Avenida Getúlio Vargas 333, 25651-075 Petrópolis, RJ, Brazil.

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Abstract. Weighted interior penalty discontinuous Galerkin method is developed to solve the two-dimensional non-equilibrium radiation diffusion equation on unstructured mesh. There are three weights including the arithmetic, the harmonic, and the geometric weight in the weighted discontinuous Galerkin scheme. For the time discretization, we treat the nonlinear diffusion coefficients explicitly, and apply the semi-implicit integration factor method to the nonlinear ordinary differential equations arising from discontinuous Galerkin spatial discretization. The semi-implicit integration factor method can not only avoid severe timestep limits, but also takes advantage of the local property of DG methods by which small sized nonlinear algebraic systems are solved element by element with the exact Newton iteration method. Numerical results are presented to demonstrate the validity of discontinuous Galerkin method for high nonlinear and tightly coupled radiation diffusion equation.

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1 Introduction

Non-equilibrium radiation diffusion systems have been used to simulate problems in inertial confinement fusion, Z-pinch experiments, and astrophysical problems [1–3]. From

*Corresponding author. *Email addresses:* rpzhang@lncc.br (R. Zhang), yuxj@iapcm.ac.cn (X. Yu), jiang@lncc.br (J. Zhu), aloc@lncc.br (A. Loula), cui_xia@iapcm.ac.cn (X. Cui)

the standpoint of partial differential equations, these systems are highly nonlinear and tightly coupled and exhibit multiple time and space scales.

In recent years, a great deal of literatures gave various numerical algorithms for the non-equilibrium radiation diffusion systems. Mostly of them are finite volume method. Knoll et al. have studied this problem in a series of papers [4–8]. Sheng et al. constructed a monotone finite volume scheme on distorted meshes for multimaterial non-equilibrium radiation diffusion equations [9], then applied the Picard iteration to the nonlinear algebraic systems. Yuan et al. presented the recent progress in numerical methods for radiation diffusion equation [10]. Their works focus on the construction of the nonlinear solver for the large nonlinear algebraic systems. Recently, Yue et al. proposed Picard-Newton iterative method to avoid the low efficiency of Picard iteration [11].

The finite element methods were also applied to the radiation diffusion systems. Mavriplis discretized the radiation diffusion systems by standard finite element method and employed the multigrid method to the nonlinear systems [12]. Kang presented P_1 nonconforming finite element for the radiation equation [13]. However, whether the finite element method or the nonconforming finite element method, they are not “local” method because the load vector will involve the nearest neighbor stencil. Their methods comprise a non-linear solver which is used to solve the non-linear equations directly and a linear solver which is used to solve the linear system arising from the linearization of the non-linear system.

In this paper, we present a weighted discontinuous Galerkin (DG) method for numerically solving radiation diffusion equation. Since their introduction over 30 years ago [14], DG methods have emerged as an attractive tool in various fields because of the flexibility for arbitrarily unstructured meshes, suitability for hp-adaptive implementation, and high parallelizability. For diffusion problems, various DG methods have been analyzed, including local discontinuous Galerkin method (LDG) [15], the diffusive generalized Riemann problem DG method (dGRP) [16, 17], the direct discontinuous Galerkin method (DDG) [18, 19], and interior penalty (IP) method [20]. Recently, Ern et al. [21] proposed weighted interior penalty (WIP) method for advection-diffusion equations with discontinuous diffusivity. The numerical flux in WIP method depends on the harmonic average weight instead of original arithmetic average. Cai et al. proposed three numerical fluxes based on the arithmetic, the harmonic, and the geometric average weight for elliptic interface problems [22]. In this paper, we evaluate the diffusion coefficients as constants on the element center and then construct weighted interior penalty method based on the above three weights.

For the time discretization, the implicit time integration methods rather than explicit methods, are often used for non-equilibrium radiation diffusion equations which have strong nonlinear transients. Previous researches include Knoll et al. [4] who compared three time integration methods: the semi-implicit method, the fully implicit Picard method, and the Newton-Krylov method. Lowrie [23] considered Beam and Warming scheme, Crank-Nicolson scheme, and predictor-corrector scheme etc. and compared the efficiency and accuracy of these methods. Ref. [24] studied linearly implicit and implicitly