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Numerical Method of Fabric Dynamics Using Front Tracking and Spring Model

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Abstract. We use front tracking data structures and functions to model the dynamic evolution of fabric surface. We represent the fabric surface by a triangulated mesh with preset equilibrium side length. The stretching and wrinkling of the surface are modeled by the mass-spring system. The external driving force is added to the fabric motion through the "Impulse method" which computes the velocity of the point mass by superposition of momentum. The mass-spring system is a nonlinear ODE system. Added by the numerical and computational analysis, we show that the spring system has an upper bound of the eigen frequency. We analyzed the system by considering two spring models and we proved in one case that all eigenvalues are imaginary and there exists an upper bound for the eigen-frequency. This upper bound plays an important role in determining the numerical stability and accuracy of the ODE system. Based on this analysis, we analyzed the numerical accuracy and stability of the nonlinear spring mass system for fabric surface and its tangential and normal motion. We used the fourth order Runge-Kutta method to solve the ODE system and showed that the time step is linearly dependent on the mesh size for the system.

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1 Introduction

Fabric material belongs to flexible objects and is more difficult to model than rigid objects. Accurate coupling of the fabric material with airflow is even more challenging. However modeling of its dynamic motion is demanded in both animation industry and engineering science. A fabric surface can be considered as a membrane which is an idealized two dimensional manifold for which forces needed to bend it are negligible when compared with forces needed to stretch and compress it. For such surface, the spring model on a triangulated mesh is a good mathematical approximation. Simulation of fabric dynamics through computational method has applications in both computer graphics and engineering. The textile and fashion industry invites computer tools that can realistically generate the shape of a cloth dressing. Scientific applications include modeling of cell skin and soft tissues. Our motivation started from the numerical study of the parachute system and its coupling with the airflow.

Many authors have contributed to the modeling of cloth and fabric surface. Terzopoulos and Fleischer [24–26] proposed continuous model for the deformable objects. Aono *et al*. [2,3] used the Tchebychev net cloth model to simulate a sheet of woven cloth composites in which they presented two algorithms, a finite difference method for the Tchebychev net and the algorithm for fitting a given 2D broadcloth composite ply to a given 3D curved surface represented by a NURBS surface. Late in 1990's and 2000's particle method gained popularity due to its intuitiveness and simplicity. Breen *et al*. [9,10], presented a particle-based model capable of being tuned to reproduce the static draping behavior of specific kinds of woven cloth. Eberhardt *et al*. [17,18] extended the model and introduced techniques to model measured force data exactly and thus cloth-specific properties. They also extended the particle system to model air resistance. Choi and Ko [15] also used particle spring model, our model is base on their work.

On the physics based modeling, Platt and Barr [20] showed how to use mathematical constraint methods based on physics and on optimization theory to create controlled, realistic animation of physically-based flexible models. Carignan *et al*. [14] discussed the use of physics-based models for animating clothes on synthetic actors in motion. Provot [22] described a physically-based model for animating cloth objects, derived from elastically deformable models, and improved order to take into account the non-elastic properties of woven fabrics. Volino *et al*. [28] presented an efficient set of techniques that simulates any kind of deformable surface in various mechanical situations. Other applications of spring model include Selle *et al*. [1] in their application to hair modeling, Terzopoulos and Waters [27] on the modeling of cutaneous tissue, subcutaneous tissue and muscle layer, and Terzopoulos *et al*. [26] modeling of deformable curves and surfaces using the elasticity theory. It should be mentioned none of the above papers has discussed the spectrum and upper bound of the frequencies and its crucial role in the oscillatory motion of the spring system.

In this paper, we follow the general idea of the particle and spring mass method for the modeling and simulation of the cloth stretching and draping. Our main focus is on