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Kinetic Energy Preserving and Entropy Stable Finite Volume Schemes for Compressible Euler and Navier-Stokes Equations

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Abstract. Centered numerical fluxes can be constructed for compressible Euler equations which preserve kinetic energy in the semi-discrete finite volume scheme. The essential feature is that the momentum flux should be of the form $f^m_{j+\frac{1}{2}} = \tilde{p}_{j+\frac{1}{2}} + \overline{u}_{j+\frac{1}{2}} f^\rho_{j+\frac{1}{2}}$ where $\overline{u}_{j+\frac{1}{2}} = (u_j + u_{j+1})/2$ and $\tilde{p}_{j+\frac{1}{2}}, f^\rho_{j+\frac{1}{2}}$ are *any* consistent approximations to the pressure and the mass flux. This scheme thus leaves most terms in the numerical flux unspecified and various authors have used simple averaging. Here we enforce approximate or exact entropy consistency which leads to a unique choice of all the terms in the numerical fluxes. As a consequence novel entropy conservative flux that also preserves kinetic energy for the semi-discrete finite volume scheme has been proposed. These fluxes are centered and some dissipation has to be added if shocks are present or if the mesh is coarse. We construct scalar artificial dissipation terms which are kinetic energy stable and satisfy approximate/exact entropy condition. Secondly, we use entropy-variable based matrix dissipation flux which leads to kinetic energy and entropy stable schemes. These schemes are shown to be free of entropy violating solutions unlike the original Roe scheme. For hypersonic flows a blended scheme is proposed which gives carbuncle free solutions for blunt body flows. Numerical results for Euler and Navier-Stokes equations are presented to demonstrate the performance of the different schemes.

AMS subject classifications: 65M08, 65M12

Key words: Euler equation, Navier-Stokes equation, finite volume method, kinetic energy preservation, entropy conservation.

1 Introduction

The numerical solution of compressible Euler and Navier-Stokes (NS) equations by the finite volume method is now a routine task in many industries, see [29] for a good review

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of numerical approaches. Due to their non-linear hyperbolic nature, solutions of Euler equations can be discontinuous with the presence of shocks or contact discontinuities. Discontinuous solutions must necessarily satisfy the Rankine-Hugoniot jump conditions which are a consequence of conservation. However it is well known that such solutions can be still non-unique and an additional entropy condition has to be imposed in order to select the unique weak solution. In the case of Euler equations, there is a natural entropy condition which comes from the entropy condition in thermodynamics which must also be satisfied by the numerical scheme. Additionally other global balance equations like that for the total kinetic energy must also be consistently approximated by the numerical solutions. The finite volume method requires the computation of the inviscid and viscous fluxes across the boundaries of the finite volumes. The design of these fluxes must incorporate the properties of the Euler/NS equations like entropy condition and kinetic energy preservation. There exists a vast library of numerical flux functions for the Euler equations and some of these like the Godunov scheme and kinetic scheme can be shown to satisfy the entropy condition. The popular Roe scheme [20] does not satisfy the entropy condition and can give rise to entropy violating shocks near sonic points. Various entropy fixes for Roe scheme have been proposed which involve preventing the numerical dissipation from vanishing at sonic points. Osher-type schemes which are similar to Roe scheme have been constructed which satisfy entropy condition [3]. Flux splitting schemes like the AUSM family [22] also satisfy the entropy condition. Tadmor [26] proposed the idea of *entropy conservative* numerical fluxes which can then be combined with some dissipation terms using entropy variables to obtain a scheme that respects the entropy condition, i.e., the scheme must produce entropy in accordance with the second law of thermodynamics. However some of these entropy conservative numerical fluxes have to be computed with quadrature rules since the integrals involved in the definition of the flux cannot be evaluated explicitly. For the Euler equations, Roe proposed explicit entropy conservative numerical fluxes [10,21] which are augmented by Roe-type dissipation terms using entropy variables. These schemes do not suffer from entropy violating solutions that are observed in the original Roe scheme. However for strong shocks, even the first order schemes can produce oscillations indicating that the amount of numerical dissipation is not sufficient. Roe [10] proposed modifying the eigenvalues of the dissipation matrix which lead to non-oscillatory solutions. The modification of the eigenvalues is such that the amount of entropy production is of the correct order of magnitude for weak shocks. The availability of cheap entropy conservative fluxes allows us to use the procedure of [15] to develop high order accurate entropy conservative schemes. Matrix dissipation can be added following the ENO procedure of [4] to develop arbitrarily high order accurate entropy stable schemes for the Euler equations on structured grids. However even entropy satisfying schemes which resolve contact waves accurately can give rise to some strange effects like the carbuncle phenomenon and shock instability problem for which additional fixes have to be applied [31].

Faithful representation of kinetic energy evolution is another desirable property of a numerical scheme [12]. This is important for direct numerical simulation of turbulent