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Gaskinetic Solutions for High Knudsen Number Planar Jet Impingement Flows

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Abstract. This paper presents a gaskinetic study and analytical results on high speed rarefied gas flows from a planar exit. The beginning of this paper reviews the results for planar free jet expanding into a vacuum, followed by an investigation of jet impingement on normally set plates with either a diffuse or a specular surface. Presented results include exact solutions for flowfield and surface properties. Numerical simulations with the direct simulation Monte Carlo method were performed to validate these analytical results, and good agreement with this is obtained for flows at high Knudsen numbers. These highly rarefied jet and jet impingement results can provide references for real jet and jet impingement flows.

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Key words: Gaskinetic theory, jet, jet impingement, Monte Carlo method.

1 Introduction

Gaseous jets expanding into a vacuum and jet impingement on a normally set plate are two fundamental fluid dynamic problems with numerous applications in engineering, physics, chemistry and other disciplines. As the counterpart to the continuum flow situation, highly rarefied jet and jet impingement flows provide one bounding limit with insights to many problems by solely including molecular movement. In many applications, the contribution from particle collisions is insignificant. One important application is the atomic/molecular beam [1,2] which is a crucial tool that leads to many extremely important scientific discoveries. Other important applications include materials processing inside vacuum chambers [3] and rocket plume effects in aerospace engineering [4,5].

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As the most important signature, rocket plume is widely used for inferred radar detections and performance evaluations. Due to the importance, many communities have been investigating rarefied gaseous jet and jet impingement flows for decades.

This paper presents some solutions for highly rarefied, or collisionless, planar jet and jet impingement flows. Section 2 reviews some background; Section 3 presents the exact solutions for the problem of high Knudsen number planar free jet into a vacuum; Section 4 shows the exact solutions for rarefied two dimensional jet impingement on a normally set flat plate with a diffuse surface; Section 5 reports some exact solutions for planar rarefied gaseous jet impingement on a specular flat plate; and Section 6 includes comparisons of the exact analytical solutions and direct simulation Monte Carlo (DSMC) [6] simulation results of rarefied impingement flows. The last section summarizes this paper with a few conclusions.

2 Background

Compressible flows can usually be summarized into four categories by the definition of the Knudsen number (Kn), which is related with the Mach (Ma) and Reynolds (Re) numbers [6–8]:

$$Kn = \frac{\lambda_0}{L} = \frac{1}{\sqrt{2}\pi d^2 n_0 L} \sim \frac{Ma}{Re}, \tag{2.1}$$

where λ_0 is the molecular mean free path, L is a characteristic length, d is the molecular diameter, and n_0 is the gas number density at the nozzle exit. These four regimes are: continuum $(0 < Kn < 0.01)$, velocity slip and temperature jump $(0.01 < Kn < 0.1)$, transitional $(0.1 < Kn < 10)$, and free molecular (or collisionless) $(10 < Kn)$. This paper discusses two flows in the free molecular regime, we choose the duct width as the characteristic length L. For the numerical validations at the end of this paper, we choose $Kn = 100$ and use Eq. (2.1) to determine the number density n_0 at the nozzle exit.

This paper focuses on exact solutions for the problems of rarefied planar jet expanding into a vacuum and jet impingement on a normally set plate with a diffused or a specular surface. A diffused reflection occurs when a particle collides at a surface, and it bounces back reversely and uniformly inside the solid angle on the other side of the local tangent plane. For a planar plate surface, the solid angle forms a span of π. For a specular reflection case, the reflected particle's normal momentum is reversed while the tangent momentum maintains unchanged.

For rarefied jet and jet impingement flows, there are many studies based on continuum theories, for example, the Navier-Stokes equations, boundary layer theory, characteristic lines, and Prandtl-Meyer flows [9]. For the high Knudsen number regime, there are many numerical and experimental studies and reports for the complete flowfield and surface properties. Most of the past studies adopted some simplifications. Noller [10] proposed a solid angle treatment to consider the nozzle exit geometry and obtained the plume density field expressed with integrations over the solid angle. Kogan [11] dis-