Commun. Comput. Phys.
doi: 10.4208/cicp.030612.010313a

Vol. **14**, No. 5, pp. 1304-1321
November 2013

Robust and Quality Boundary Constrained Tetrahedral Mesh Generation

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Received 3 June 2012; Accepted (in revised version) 1 March 2013

Available online 13 June 2013

Abstract. A novel method for boundary constrained tetrahedral mesh generation is proposed based on Advancing Front Technique (AFT) and conforming Delaunay triangulation. Given a triangulated surface mesh, AFT is firstly applied to mesh several layers of elements adjacent to the boundary. The rest of the domain is then meshed by the conforming Delaunay triangulation. The non-conformal interface between two parts of meshes are adjusted. Mesh refinement and mesh optimization are then preformed to obtain a more reasonable-sized mesh with better quality. Robustness and quality of the proposed method is shown. Convergence proof of each stage as well as the whole algorithm is provided. Various numerical examples are included as well as the quality of the meshes.

AMS subject classifications: 65L50, 65M50, 65N50, 65D18

Key words: Unstructured tetrahedral mesh, Advancing Front Technique, conforming Delaunay triangulation, boundary constrained, mesh refinement, mesh optimization.

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1 Introduction

Automatic tetrahedral mesh generation is always an important yet challenging part in many scientific and engineering computation problems. As the preprocessor of computation or analysis, it plays a significant role in discretizing the domain of interest. Extensive research has been conducted in tetrahedral meshing field and tremendous advances have been made on both theoretical analysis and robust implementation. Most existing tetrahedral meshing techniques can be categorized into three groups: 1. Spatial decomposition methods, i.e octree-based methods [1]; 2. Advancing front triangulation methods [2,3]; 3. Delaunay triangulation based methods [4–8].

The input for most mesh generators is a surface triangulation, which bounds the domain of interest. The desirable generated mesh shall conform with the input triangulation in such a way that the geometric constraints in the triangulation, i.e. edges and faces, are preserved in the generated volumetric mesh. Usually there are two manners to preserve the constraints. The first one is *boundary conforming way*, in which the constraints can be kept as the concatenation of edges/faces. This manner could possibly introduce extra points on the constraints. The second manner is *boundary constrained way*, in which no extra point on the constraints is allowed in the result mesh. It is required that all constraints are preserved in their original and integral forms. Though both manners are acceptable in most computational fields, a mesh constrained to boundary is often preferred due to two reasons. (a) Boundary constrained way respects the input sizing specification and no extra point on constraints modifies the sizing function; (b) it offers more robustness than the conforming way when meshes from two connected regions are merged together. A efficiency and robust boundary constrained mesh generator is also our goal. Next we simply describe how three groups of methods obtain a boundary constrained mesh as well as their advantages and disadvantages.

1. In spatial decomposition methods, a series of recursively variable sized cubes described by an octree is used to approximate the domain of interest. The input for most octree methods is the geometric representation without boundary constraints. The boundary triangulation is generated by intersecting the leaf cubes of octree with the geometric boundary. It is difficult for octree methods to generate boundary constrained meshes.

2. In advancing front methods, mesh is generated element by element along the boundary of existing mesh, i.e. *front*. The triangle on the *front* is taken as base and an apex is selected or created to construct a tetrahedron, the rule of which includes that the new tetrahedron shall not destroy the existing mesh. This construction rule naturally guarantees that the input surface triangulation would not be destroyed. The generated mesh will be constrained to boundary inherently. Besides, advancing front provides good qualities for the elements near the boundary, which are favored by many applications, such as computational fluids. However, the efficiency and the robustness of advancing front is always questionable in the community. It