Pressure Distribution of the Gaseous Flow in Microchannel: A Lattice Boltzmann Study

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Abstract. In this paper the pressure distribution of the gaseous flow in a microchannel is studied via a lattice Boltzmann equation (LBE) method. With effective relaxation times and a generalized second order slip boundary condition, the LBE can be used to simulate rarefied gas flows from slip to transition regimes. The Knudsen minimum phenomena of mass flow rate in the pressure driven flow is also investigated. The effects of Knudsen number (rarefaction effect), pressure ratio and aspect ratio (compression effect) on the pressure distribution are analyzed. It is found the rarefaction effect tends to the curvature of the nonlinear pressure distribution, while the compression effect tends to enhance its nonlinearity. The combined effects lead to a local minimum of the pressure deviation. Furthermore, it is also found that the relationship between the pressure deviation and the aspect ratio follows a pow-law.

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1 Introduction

Pressure driven gaseous flows in microchannels are getting more attention nowadays due to their key role in the Micro-Electro-Mechanical-System (MEMS) [1–3]. Usually, a microchannel flow is characterized by the Knudsen number $Kn = \lambda / L$, where $\lambda$ is the mean-free-path (MFP) of the gas and $L$ is the channel height. According to the value of $Kn$, the flow can be classified into four types, i.e., continuum flow ($Kn < 0.001$), slip flow ($0.001 \leq Kn < 0.1$), transition flow ($0.1 \leq Kn < 10$), and free-molecular flow ($Kn \geq 10$). In such flows, the gas density changes along the channel and $Kn$ vary in a wide range for

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a long channel. Consequently, it is rather a challenging task to simulate such multi-scale flows.

Previously, a variety of studies have been conducted to simulate gaseous flows in microchannels using different methods. Arkilic et al. obtained an analytical solution of the pressure driven flows in long channels based on the compressible Navier-Stokes (N-S) equations with a first-order slip boundary condition [4]. Jang et al. gained a more accurate analytical solution by considering the geometry and temperature [5]. Since the N-S equations together with the slip boundary condition do not work beyond the slip regime, the results are only valid for slip flows. Shen et al. applied the Information-Preservation Direct Simulation Monte Carlo (IP-DSMC) method to simulate the microchannel flows in both slip and transitional regimes, where the mass flow and pressure distribution were reported [6]. Maurer et al. investigated the effect of the slip coefficient on the slip regime based on the N-S equations with a second order slip boundary condition [7]. Varoutis et al. presented some computations by lattice Boltzmann equation (LBE) and experimental results of the gaseous flows through long channels with triangular and trapezoidal cross sections [8]. On the other hand, Ohwada et al. explored the channel flow [9] by solving the linearized Boltzmann equation under the assumption of small pressure gradient.

However, most of the available studies concentrate on the gas velocities and mass flow rates, and very few attention has been paid to the distribution of pressure under different conditions. Due to the combined rarefaction and compression effects in microscale gaseous flows, the pressure distribution in the channel is usually nonlinear. In this work, we will focus on this topic to understand how the pressure distribution is influenced by the flow parameters.

The rest of the paper is organized as follows. The physical problem is described in Section 2, and in Section 3 we briefly introduce the numerical method. Numerical results and discussions are then provided in Section 4, and finally a summary is given in Section 5.

2 Problem description

The problem considered is a pressure-driven gas flow in a two-dimensional (2D) microchannel with length \( L \) and height \( H \) as shown in Fig. 1, two walls locate at \( y = 0 \) and \( y = H \), respectively. The aspect ratio is defined as \( \varepsilon = H/L \). The pressure at the inlet and outlet are fixed at \( P_{in} \) and \( P_{out} \), respectively, and the pressure ratio is denoted by \( \theta_p = P_{in}/P_{out} \).

In slip regime, the flow can be described by the compressible Navier-Stokes equations, supplemented by a slip velocity,

\[ u_{wall} = \sigma \frac{2}{\sigma} \text{Kn} \frac{\partial u}{\partial y}|_{wall} \]  

(2.1)

where \( u \) is the tangential velocity, \( y \) is the spanwise direction, \( \sigma \) is the tangential momentum accommodation. Arkilic et al. [4] solved the governing equations with a perturbation