

## An Efficient Numerical Model for Immiscible Two-Phase Flow in Fractured Karst Reservoirs

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**Abstract.** Numerical simulation of two-phase flow in fractured karst reservoirs is still a challenging issue. The triple-porosity model is the major approach up to now. However, the triple-continuum assumption in this model is unacceptable for many cases. In the present work, an efficient numerical model has been developed for immiscible two-phase flow in fractured karst reservoirs based on the idea of equivalent continuum representation. First, based on the discrete fracture-vug model and homogenization theory, the effective absolute permeability tensors for each grid blocks are calculated. And then an analytical procedure to obtain a pseudo relative permeability curves for a grid block containing fractures and cavities has been successfully implemented. Next, a full-tensor simulator has been designed based on a hybrid numerical method (combining mixed finite element method and finite volume method). A simple fracture system has been used to demonstrate the validity of our method. At last, we have used the fracture and cavity statistics data from TAHE outcrops in west China, effective permeability values and other parameters from our code, and an equivalent continuum simulator to calculate the water flooding profiles for more realistic systems.

**AMS subject classifications:** 35B27, 35Q30, 76S05

**Key words:** Fractured karst reservoirs, effective permeability tensor, discrete fracture-vug network model, two-phase flow, full-tensor simulator.

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## 1 Introduction

Fractured karst reservoirs are composed of porous material, which contains cavities (or vugs) and fractures on multiple scales and throughout the entire rock formation [1]. The presence of fractures and cavities, often relatively large void spaces, affects the flow paths

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in the medium and should be accurately accounted for in the numerical model. Modeling flow and transport processes in fractured karst reservoirs is still a challenging problem and the main difficulty is the co-existence of porous flow and free-fluid flow.

In the past 5 years, there are some scholars and engineers have paid much attention to the modeling single-phase flow in fractured karst carbonate reservoirs [2–9]. And their researches mainly focus on the equivalent absolute permeability analysis and the effect of the fractures and cavities based on homogenization of Stokes-Darcy equations or Stokes-Brinkman equations. Recently, in [10] and [11] Z.-Q. Huang and J. Yao et al. proposed a novel flow model named discrete fracture-vug network model (DFVN), which is efficient for single-phase flow in fractured karst carbonate reservoirs. The similar conceptual model also has been proposed by G. Qin et al. [12]. However, the multi-phase and two-phase flow based on discrete fracture-vug network model is still a challenging and open problem, since the governing equations of the free and porous flows are quite different from each other and involve both microscopic and macroscopic formulations.

As observed in carbonate formation, three porosity types (matrix, fractures, and cavities) are typically presented in naturally fractured karst reservoirs. These fractures and cavities distribute irregularly and vary in size, from microscopic to macroscopic. And the fractures and cavities are generally connected to form a fracture-cavity network [10, 11]. Drawing on the concept of dual-porosity model for fractured reservoirs, Y. S. Wu et al. proposed a triple-porosity model to study the flow and transport behaviors in fractured karst reservoirs [13–15]. And a field application was conducted in TAHE oilfield in west China [13]. However, the triple-continuum assumption is unacceptable for many cases.

In the presented paper, we will describe another alternative numerical model for immiscible two-phase flow in fractured karst reservoirs. There are three key steps in our approach. First, based on the discrete fracture-vug model and homogenization theory, the equivalent absolute permeability tensors (i.e., the ability to transmit fluids) for each grid block are calculated. Within this step only a steady-state single-phase flow model is used. In the next step, a pseudo relative permeability curves for each grid block containing fractures and cavities are obtained in an analytical procedure. The procedure is based on a fixed sequence of oil displacement from grid cells containing fractures and cavities: the volume of the fractures and cavities within a grid block is assumed to fill with water before the matrix volume of this grid-block is flooded (called the preferential flow assumption). This means that the fracture-vug network is the preferential flow path in grid blocks. In fractured karst reservoirs this is the main displacement sequence in waterflooding, at least for TAHE oilfield in west China. In the third step, a full-tensor reservoir simulator should be designed for this equivalent continuum model.

In the following Section we first outline the basis of our model and describe how we applied the discrete fracture-vug network to obtain the equivalent absolute permeability tensor. In Section 2.2 we describe the use of an analytical method to calculate the pseudo relative permeability curves in a grid block containing fracture-cavity network. In Section 3, a full tensor simulator has been designed based on a hybrid numerical method, in which the pressure equation is discretized using mixed finite element method and the