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Sound Propagation Properties of the Discrete-Velocity Boltzmann Equation

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Abstract. As the numerical resolution is increased and the discretisation error decreases, the lattice Boltzmann method tends towards the discrete-velocity Boltzmann equation (DVBE). An expression for the propagation properties of plane sound waves is found for this equation. This expression is compared to similar ones from the Navier-Stokes and Burnett models, and is found to be closest to the latter. The anisotropy of sound propagation with the DVBE is examined using a two-dimensional velocity set. It is found that both the anisotropy and the deviation between the models is negligible if the Knudsen number is less than 1 by at least an order of magnitude.

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1 Introduction

The lattice Boltzmann method (LBM) is a fairly recent development in computational fluid dynamics (CFD). While traditional CFD methods are based on discretising the conservation equations of the continuum model, the LBM is based on discretising the Boltzmann equation from the kinetic theory of gases. The Boltzmann equation describes how distributions of particles in a gas propagate and collide, thus giving a more detailed picture than the continuum model. It can be shown that the total behaviour of these particle distributions at long time scales corresponds with the conservation equations of the continuum model [1].

Although the lattice Boltzmann method can be used to simulate weakly compressible flow [2], current research has largely been confined to incompressible flow. However, in

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the last few years several articles have been published on applying the LBM for computational aeroacoustics (CAA) [3, 4], i.e. for simulating generation of sound waves in unsteady flow. This subject is based on the theory of aeroacoustics first developed by Lighthill [5].

In some cases the generated sound has a strong feedback interaction with the fluid flow, as in the problem of tone generation in corrugated pipes [6]. These cases must be studied using a compressible flow simulation. As the LBM is more straightforward to implement and more parallelisable than traditional compressible CFD methods, it could be a useful supplement to traditional CAA methods.

However, the propagation of sound waves for the LBM has not yet been sufficiently studied. A previous article by this author looked at the case of plane sound waves in the LBM [7], and showed disagreement between the LBM and Navier-Stokes even in the limit of no discretisation error. The goal of the present article is twofold: To explain this disagreement, and to further examine the behaviour of the LBM in this limit. The focus here will be narrower than in the previous article; this article will only look at absorption and dispersion of spatially damped plane sound waves.

The limit of no discretisation error is an important one; if a numerical method does not behave correctly in this limit it is inconsistent, and its behaviour can not necessarily be improved by improving the numerical resolution.

In Section 2, the basics of damped plane sound waves are explained. Section 3 derives an analytic expression for the propagation of these sound waves from the discrete-velocity Boltzmann equation (DVBE), which corresponds to the aforementioned limit of the LBM. This is compared in Section 4 with similar expressions from other models. Section 5 extends the derivation from Section 3 to two dimensions, and examines the isotropy properties of the DVBE.

2 Damped sound waves

In a sound wave, the density ρ, particle velocity $\mathbf{u}$, and pressure p oscillate around an equilibrium state. We assume here that the oscillations are infinitesimal monofrequency plane waves propagating in the $+x$-direction, and write them in phasor form,

$$\begin{bmatrix} \hat{\rho}(x,t) \\ \hat{u}(x,t) \\ \hat{p}(x,t) \end{bmatrix} = \begin{bmatrix} \rho_0 \\ 0 \\ p_0 \end{bmatrix} + \begin{bmatrix} \hat{\rho}' \\ \hat{u}' \\ \hat{p}' \end{bmatrix} \mathrm{e}^{\mathrm{i}(\hat{\omega} t - \hat{k} x)}. \tag{2.1}$$

Throughout this article, hats indicate complex numbers and primes are used for infinitesimally small oscillation amplitudes.

If we split the angular frequency $\hat{\omega}$ and wavenumber $\hat{k}$ into real and imaginary parts,

$$\hat{\omega} = \omega_{\mathrm{r}} + \mathrm{i}\alpha_t, \qquad \hat{k} = k_{\mathrm{r}} - \mathrm{i}\alpha_x, \tag{2.2}$$