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A *h*-Adaptive Algorithm Using Residual Error Estimates for Fluid Flows

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Abstract. Algorithms for adaptive mesh refinement using a residual error estimator are proposed for fluid flow problems in a finite volume framework. The residual error estimator, referred to as the $\Re$-parameter is used to derive refinement and coarsening criteria for the adaptive algorithms. An adaptive strategy based on the $\Re$-parameter is proposed for continuous flows, while a hybrid adaptive algorithm employing a combination of error indicators and the $\Re$-parameter is developed for discontinuous flows. Numerical experiments for inviscid and viscous flows on different grid topologies demonstrate the effectiveness of the proposed algorithms on arbitrary polygonal grids.

AMS subject classifications: 65M08, 65M50, 65Z05, 76N15

Key words: Adaptive refinement, error estimator, residual, error indicator.

1 Introduction

The primary focus in numerical simulations of practical engineering problems which invariably involve complexities in both geometry and flow, is to obtain accurate numerical solutions at shorter turnaround times. Unfortunately, the computational effort in obtaining the solutions and the solution accuracy are at conflict: finer meshes lead to accurate solutions at greater computational effort, while coarse meshes involve lower computational effort but result in inaccurate solutions. *Adaptive Mesh Refinement* (AMR) algorithms constitute a class of computational methods that strike the right balance between numerical solution accuracy and the associated computational effort and provide

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a viable and economical approach for simulation of complex flow problems. AMR algorithms have been employed by several researchers for a variety of flow problems ranging from shock hydrodynamics [1] and compressible flows [2] to multi-phase flows [3] and astrophysical applications [4].

Sensors employed in the AMR algorithm can be broadly classified as *Error Indicators* and *Error Estimators*. Several adaptive algorithms in the past have relied on error indicators, mainly due to their ability to precisely detect flow phenomena. Some of the popular error indicators include curl and divergence of velocity [5] and divided differences in density [6, 7] for shocked flows and vorticity gradient [8] for vortex dominated flows. Error indicators, however, provide no information on error levels in the domain which can result in a termination criterion for refinement. Consequently, all indicator based adaptive refinement algorithms resort to heuristic considerations involving user-defined parameters that are problem-dependent. Error estimators, on the other hand, provide a reasonable estimate of some error distribution in the domain and can therefore be exploited to derive a suitable termination criterion. Error estimators can be broadly classified as global error based estimators, adjoint based estimators and residual estimators. Since the focus of this work is on residual error estimation, we shall discuss only this class of estimators in detail. For a review on other error estimators, refer to [9, 10] and references therein.

Residual error estimation have been extensively studied in the finite element framework [11], though only few studies in the context of finite volume framework are reported in literature. Particularly in the context of finite volume computations, the residual error estimators provide an estimate of the local truncation error, which is the extent to which the discrete algebraic equation differs from the partial differential equation it models. Jasak and Gosman [12] proposed the element residual estimate for finite volume discretisations. Aftsomis and Berger [13] compute the local truncation error using an approximation of the exact solution. Hay and Visonneau [14] have proposed the use of a higher order reconstruction operator to compute the residual, while Karni and Kurganov [15] have introduced the concept of weak local residuals to compute the truncation error. Roy and Sinclair [16] have developed an interesting approach where the an analytical expression for the numerical solution obtained using curve-fitting techniques is employed to estimate the truncation error. The authors in one of their earlier works have also proposed an estimator called the $\Re$-parameter which is a novel and generic approach to residual estimation in a finite volume framework [17–20].

There are several merits to the use of $\Re$-parameter in an adaptive strategy as amply demonstrated in reference [17]: (i) The computation of $\Re$-parameter is cheap and does not require any approximation to the exact solution (in contrast to the estimators of the global error). (ii) It is possible to obtain theoretically the rate of fall of the $\Re$-parameter with grid refinement and this is not so obvious in the case of global errors. (iii) Apart from this, the $\Re$-parameter marks the source of errors in problems involving error transport and therefore offer better control over the global error (the ultimate objective of any adaptive calculation) as compared to global error based mesh adaptation. (iv) One of the