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Energy Conserving Lattice Boltzmann Models for Incompressible Flow Simulations

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Abstract. In this paper, we highlight the benefits resulting from imposing energy-conserving equilibria in entropic lattice Boltzmann models for isothermal flows. The advantages are documented through a series of numerical simulations, such as Taylor-Green vortices, cavity flow and flow past a sphere.

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1 Introduction

In the last decade, mesoscale algorithms such as lattice Boltzmann models (LBM), Dissipative Particle Dynamics (DPD) and multi-particle collision dynamics, have attracted increasing interest in the framework of computational fluid dynamics (see, e.g., [1–15]). This success story is remarkable from the theoretical point of view too, as continued effort in this field has succeeded in establishing the existence of a self-consistent underlying micro-dynamics behind the mesoscopic formulation.

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For example, in the case of LBM, the importance of formulating discrete kinetic models in compliance with the H-theorem, is by now fully appreciated [3, 16–21]. In fact, an exact lattice analog of the continuous Maxwell-Boltzmann distribution was derived from a discrete version of the entropy maximization principle [19,20,22]. The link between discrete thermodynamics and numerical stability and efficiency of the corresponding computational model, is also well appreciated and possible generalizations towards more microscopic formulations have been explored in recent works [11, 19, 23–27].

Despite the aforementioned success of these approaches, much still needs to be understood, both from theoretical and numerical standpoint, such as efficient implementation of curved boundaries, numerical stability at very low viscosity and others.

In the present manuscript, we show that releasing a specific thermodynamic deficiency of the method, leads to a significant improvement in the quality of the simulation. More precisely, in its present popular isothermal setting, sound propagation in lattice Boltzmann [28], takes place at constant temperature, thus following Newton's definition of sound speed,

$$c_s^2 = \frac{\partial P}{\partial \rho}\bigg|_T = \frac{k_{\rm B} T_0}{m} \equiv v_0^2, \tag{1.1}$$

where v_0 is the reference thermal speed.

However, via Laplace theory, it is known that, in actual reality, sound propagation occurs via an adiabatic process, which can only be described by an energy conserving (EC) model. This automatically gives,

$$c_s^2 = \frac{\partial P}{\partial \rho}\bigg|_S = \gamma \frac{k_{\rm B} T_0}{m}, \tag{1.2}$$

where γ is the adiabatic exponent. Traditionally, this discrepancy was largely neglected in isothermal LBM simulations, with an argument that the relevant observable is the velocity field, the sound speed being just an immaterial constant. However, as we shall show in the following, this thermodynamic aspect plays a major role in determining the quality of simulation results even for isothermal flows in fully resolved domains. In other words, reproducing the correct sound speed gives rise to a much more robust numerical scheme.

The work is organized as follows. In Section 2, lattice Boltzmann model (both energy conserving and isothermal) is briefly reviewed. In Section 3, via an example we show that energy conserving model indeed manages to reproduce adiabatic sound propagation correctly. In Section 4, we compare the energy conserving model with isothermal model for the set up of Taylor-Green vortex, cavity flow and flow past a sphere. Finally, we summarize results of the study in Section 5.

2 Lattice Boltzmann method

We briefly remind the reader that, in typical LBM formulations, one works with a set of discrete populations $f = \{f_i\}$, corresponding to predefined discrete velocities $\mathbf{c}_i$ $(i =$