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Bifurcation Diversity in an Annular Pool Heated from Below: Prandtl and Biot Numbers Effects

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Abstract. In this article the instabilities appearing in a liquid layer are studied numerically by means of the linear stability method. The fluid is confined in an annular pool and is heated from below with a linear decreasing temperature profile from the inner to the outer wall. The top surface is open to the atmosphere and both lateral walls are adiabatic. Using the Rayleigh number as the only control parameter, many kind of bifurcations appear at moderately low Prandtl numbers and depending on the Biot number. Several regions on the Prandtl-Biot plane are identified, their boundaries being formed from competing solutions at codimension-two bifurcation points.

AMS subject classifications: 76R10, 35P15

Key words: Thermocapillary convection, Prandtl number, Biot number, linear stability.

1 Introduction

The problem of thermoconvective instabilities in fluid layers driven by a temperature gradient has become a classical subject in fluid mechanics [1,28]. Two different effects are responsible for the onset of motion when the temperature difference becomes larger than a certain threshold: gravity and capillary forces. When both effects are taken into account the problem is called Bénard-Marangoni (BM) convection [1]. Classically, heat is applied uniformly from below [1] where the conductive solution becomes unstable for increasing temperature gradients. A more general set-up may be considered which includes

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thermoconvective instabilities by imposing a basic dynamic flow through non-zero horizontal temperature gradients, either in rectangular geometries [3,6,10,13,15,17,21,22,28] or in cylindrical and annular geometries [7, 8, 12, 13, 19]. In particular, references [8, 14] and Garnier's PhD thesis [9] include a revision of the flow configuration found in this sort of problems. It is also worthy mentioning the experimental work of Schwabe et al. [24], performed in low gravity conditions.

These studies are characterized by a set of dimensionless numbers:

1. Rayleigh number, $\mathrm{Ra}=g\alpha\Delta T d^4/\kappa\nu$: Representative of the buoyancy effect.
2. Marangoni number, $\mathrm{Ma}=\gamma\Delta T d^2/\rho\kappa\nu$: Accounts for the surface tension effects.
3. Prandtl number, $\mathrm{Pr}=\nu/\kappa$: The ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. In this article Pr values range from 1 to 20.
4. Bond number, $\mathrm{Bo}=\mathrm{Ra}/\mathrm{Ma}=\alpha\rho d^2/\gamma$: Ratio of Rayleigh to Marangoni numbers, which is kept constant in this article.
5. Biot Number, Bi: Accounts for heat transmission between the fluid and the atmosphere. Values inside the range $[0.2-1.5]$ are explored in this article.
6. Aspect ratio, $\Gamma=\delta/d$.

Here γ stands for the rate of change of surface tension with temperature, κ is the thermal diffusivity, ν is the kinematic viscosity of the liquid, α is the thermal expansion coefficient, g is the gravitational acceleration, ΔT stands for a temperature increment and δ and d are characteristic lengths to be defined later. The reference values used are similar to those employed in [12], so that $\mathrm{Bo}\sim 70$ and buoyancy effects are dominant. In this work, Prandtl, Biot and Rayleigh numbers were supposed to be independent parameters and their effects on the solution of the problem was carefully studied.

In recent years Shi, Peng and several collaborators have studied numerically an annular geometry, [23,25–27], with a method similar to that used in [12]. The main differences between these works and the present article is that in those contributions the effects of the Biot number were not considered and that the lateral walls of the annular pool were conductive. Other approximations to this sort of problems have been proposed which make use of tools coming from functional analysis, see [20] for details and references therein, but the presence of a tangential derivative in the Marangoni condition (see next section) makes this approximation almost impracticable.

Results on this problem were obtained in [12, 13] which evidenced the importance of heat-related parameters in the development of the instabilities. In [14] the authors found that very diverse bifurcations are controlled by the Biot number and compared their solutions with the experimental results obtained by the group of Garnier [8]. The main interest of this paper is to generalized the results of these works, removing the infinite-Prandtl number approximation.

The paper is structured as follows. In the second section the formulation of the problem and the numerical method used to solve it are presented. Then, in the third section