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Additive Schwarz Preconditioners with Minimal Overlap for Triangular Spectral Elements

Yuen-Yick Kwan*

Center for Computational Science, Tulane University, New Orleans, LA 70118, USA.

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Abstract. The additive Schwarz preconditioner with minimal overlap is extended to triangular spectral elements (TSEM). The method is a generalization of the corresponding method in tensorial quadrilateral spectral elements (QSEM). The proposed preconditioners are based on partitioning the domain into overlapping subdomains, solving local problems on these subdomains and solving an additional coarse problem associated with the subdomain mesh. The results of numerical experiments show that the proposed preconditioner are robust with respect to the number of elements and are more efficient than the preconditioners with generous overlaps.

AMS subject classifications: 65F08, 65N35

Key words: Spectral elements, additive Schwarz.

1 Introduction

Spectral element method, which combines the flexibility of the low-order finite element methods and the high accuracy of the spectral methods, are popular in computational science and engineering. The efficiency of the spectral element method depends on the solution method employed to solve the resultant linear system. Since direct methods are infeasible when the number of elements get large, preconditioned iterative methods are usually used. Preconditioners can be based on an overlapping or non-overlapping domain decomposition [24, 28]. The former include the multiplicative Schwarz [18], additive Schwarz [9] and the restricted Schwarz preconditioners [6]. The multiplicative Schwarz preconditioner generally has better convergence properties than the additive counterpart, but there is no straightforward way to parallelize it. The non-overlapping

*Corresponding author. *Email address:* `tkwan@tulane.edu` (Y.-Y. Kwan)

preconditioners include Neumann-Neumann [10], Finite Element Tearing and Interconnect (FETI) [13], Balancing Domain Decomposition methods [20], Dual-Primal Finite Element Tearing and Interconnect (FETI-DP) [12], Balancing Domain Decomposition methods by Constraints (BDDC) [8] and optimized Schwarz [14, 22]. The preconditioned system from the FETI-DP and BDDC preconditioners have essentially the same spectrum [5,17,21]. Though most of these preconditioners are originally proposed for quadrilateral elements (QSEM), they can also be applied to triangular elements (TSEM). One exception is the additive Schwarz preconditioner with minimal overlap [7,19], as its construction relies heavily on the tensor-product nature of the collocation points.

In this paper, we construct additive Schwarz preconditioners with minimal overlap for TSEM. The construction is made possible by considering the Schur complement instead of the full matrix. We also compare the performance of the different preconditioners, including the Schwarz preconditioners with generous and minimal overlaps and the BDDC preconditioner. Four sets of collocation points are used in the numerical experiments, including the uniformly-distributed points, the Fekete points [2, 27], the Lobatto points [1] and the Lebesgue points [25].

The rest of the paper is organized as follows. Section 2 gives a brief review of spectral element method. Section 3 gives a brief review of the Schur complement and proves a theorem related to it. Section 4 introduces the different preconditioners and compares their computational complexities. Numerical experiments are presented in Section 5. Finally, some concluding remarks are given in Section 6.

2 Spectral element method

Consider the screened Poisson equation

$$\alpha u - \nabla^2 u = f \tag{2.1}$$

in $\Omega \subset \mathcal{R}^2$ with boundary conditions

$$u = 0 \quad \text{on } \Gamma_{\rm D}, \qquad \frac{\partial u}{\partial n} = g_{\rm N} \quad \text{on } \Gamma_{\rm N}, \tag{2.2}$$

where α is a non-negative constant and $\Gamma_{\rm D} \cup \Gamma_{\rm N} = \partial\Omega$. Let $\mathcal{T} = \{T_i : i = 1, \cdots, N\}$ be a triangulation of the domain Ω, such that

$$\bigcup_{i=1}^{N} \overline{T_i} = \overline{\Omega}, \qquad T_i \cap T_j = \emptyset, \quad \text{for } i \neq j.$$

Then the spectral element space for the solution is

$$W_p(\mathcal{T}) := \mathcal{P}_p(\mathcal{T}) \cap W, \tag{2.3}$$