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Lattice BGK Model for Incompressible Axisymmetric Flows

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Abstract. In this paper, a lattice Boltzmann BGK (LBGK) model is proposed for simulating incompressible axisymmetric flows. Unlike other existing axisymmetric lattice Boltzmann models, the present LBGK model can eliminate the compressible effects only with the small Mach number limit. Furthermore the source terms of the model are simple and contain no velocity gradients. Through the Chapman-Enskog expansion, the macroscopic equations for incompressible axisymmetric flows can be exactly recovered from the present LBGK model. Numerical simulations of the Hagen-Poiseuille flow, the pulsatile Womersley flow, the flow over a sphere, and the swirling flow in a closed cylindrical cavity are performed. The results agree well with the analytic solutions and the existing numerical or experimental data reported in some previous studies.

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Key words: Lattice BGK model, incompressible axisymmetric.

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1 Introduction

In the past few years, the lattice Boltzmann methods (LBM) originated from kinetic theory, have gained much attention in hydrodynamics [1–5]. Compared with the traditional methods (for example, finite difference method, finite element method and finite volume method), the LBM have many advantages, such as the simplicity of program, location of computation, nature parallelism and easiness in dealing with complex boundary. The

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lattice BGK (LBGK) model, as the most popular LBM, has been successfully applied to study a variety of fields such as flow in porous media, heat transfer, turbulence, blood flow, Chemical reactions, and multiphase and multicomponent flows. It is well known that when the standard LBGK model is used to simulate incompressible fluid flows, there may be compressible effects existed which might lead to some undesirable errors. In order to eliminate the compressible effects, many authors have developed some incompressible LBGK models [6–10]. Only Guo's LBGK model [10] can effectively eliminate the compressible effect induced by density variation, and the incompressible Navier-Stokes (NS) equations can be exactly recovered from this model.

Up to now, there are some LBGK models proposed for axisymmetric flows. To simulate the three-dimensional (3D) axisymmetric flows on the Cartesian coordinate system [11–13], the most direct way is to apply certain 3D LBGK models with suitable curved boundary treatment. However, it is well known that 3D axisymmetric flows are in effect 2D problem in the cylindrical coordinate system. In order to make use of the advantages of the axisymmetric properties, Halliday et al. firstly developed the LBGK model for axisymmetric flows [14]. They inserted "source" or "forcing" terms into the evolving equations so that it could recover the axisymmetric Navier-Stokes (NS) equations. Shortly after the presentation of this method, Premnath and Mukherjee extended it to multiphase flows and two-phase flows with lager density ratio respectively [15, 16]. Unfortunately, the model of Halliday et al. was found to miss some important terms relative to the radial velocity. Additionally, since this model was derived from the standard LBGK model, the compressible effect can not be eliminated. Lee et al. [17] firstly point out these limitations and developed a more accurate axisymmetric LBGK model from the incompressible LBGK model proposed by He and Luo [9]. Shortly afterwards, Reis and Phillips developed a modified model following the philosophy proposed by Halliday [18, 19]. Zhou presented a much simpler axisymmetric LBGK model in the year 2008 [20]. Through Chapman-Enskog (C-E) expansion, the added source terms in the model happened to be the additional in the governing equations for the axisymmetric flows compared with the NS equations. Recently, Chen et al. developed an incompressible D2Q5 LBGK model for axisymmetric flows based on the vorticity-stream equations [21].

Although the above axisymmetric LBGK models have been used to simulate various flows [22–30], they still have some limitations. Firstly as pointed out in [31], these models (except for the one in [21]) almost include many velocity gradients in the source terms. The discretization of these gradient terms may leads to additional errors and numerical instability. Although the model in [21] can simplify the source terms, a Poisson equation must be solved at each time step will lead to inefficient for unsteady flows. Secondly, all of these models neglect the azimuthal velocity. Finally, most of these models (except the ones in [17] and [21]) are constructed from the standard LBGK model, so they can only be viewed as artificial compressible methods for simulating incompressible axisymmetric flows. When the model proposed in [17] is used to simulate unsteady incompressible axisymmetric flows, in order to neglect the artificial compressible effect, an additional conditions, $L_x/(TC_s) \ll 1$, must be required. Furthermore, the average pressure of the