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On Computational Modelling of Strain-Hardening Material Dynamics

Philip Barton[1,*] and Evgeniy Romenski[2]

[1] *Graduate Aerospace Laboratories, California Institute of Technology, California 91125, USA.*
[2] *Sobolev Institute of Mathematics, Russian Academy of Sciences, Novosibirsk 630090, Russia.*

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Abstract. In this paper we show that entropy can be used within a functional for the stress relaxation time of solid materials to parametrise finite viscoplastic strain-hardening deformations. Through doing so the classical empirical recovery of a suitable irreversible scalar measure of work-hardening from the three-dimensional state parameters is avoided. The success of the proposed approach centres on determination of a rate-independent relation between plastic strain and entropy, which is found to be suitably simplistic such to not add any significant complexity to the final model. The result is sufficiently general to be used in combination with existing constitutive models for inelastic deformations parametrised by one-dimensional plastic strain provided the constitutive models are thermodynamically consistent. Here a model for the tangential stress relaxation time based upon established dislocation mechanics theory is calibrated for OFHC copper and subsequently integrated within a two-dimensional moving-mesh scheme. We address some of the numerical challenges that are faced in order to ensure successful implementation of the proposed model within a hydrocode. The approach is demonstrated through simulations of flyer-plate and cylinder impacts.

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Key words: Entropy, viscoplasticity, solid mechanics, relaxation time, Maxwell solid, Eulerian.

1 Introduction

It is widely recognised that physically consistent continuum models for governing strain-hardening inelastic deformations in solid materials require knowledge of certain internal variables. The irreversibility of these processes reflects the need for a suitable irreversible

*Corresponding author. *Email addresses:* `ptbarton@caltech.edu`, `phil.t.barton@gmail.com` (P. Barton), `evrom@math.nsc.ru` (E. Romenski)

measure within the material. For calculations of uniaxial loading tests, formulation of the constitutive models for the flow stress, or stress rate for viscoplastic models, in terms of the easily determinable plastic strain is adequate for this purpose. Ordinarily the principle of such calculations is to calibrate the constitutive models for inelastic deformations, through comparison of the theory with available experimental data, with the eventual intent of integrating these within a more complex model for simulations in higher space dimensions for which the uniaxial model is a subset. However, employing the plasticity models within a multi-dimensional framework is complicated since the strain then forms a potentially non-spherical rank-two tensor. Assuming the material of interest is isotropic, the frame indifferent plasticity models derived in this way employ only scalar measures of state which must thus be recovered in some way through functionals of the higher-dimensional state variables. The classical Lagrangian models that include the process of work hardening have relied upon a one-dimensional measure of plastic strain determined through integration of the equivalent plastic strain rate to acquire the Von Mises strain. Within Eulerian formulations the permissible advection of material through control volumes requires the strain history to be tracked, and the Von Mises plastic strain is typically declared as an internal parameter for which an additional transport law can be derived with the production defined on the basis of, for example, the Levy-St. Venant flow rule. In both the Lagrangian and Eulerian variants of the continuum theory the determination and use of equivalent plastic strain in models describing inelastic deformations is empirical. This paper is devoted to investigating the axiom that the thermodynamically consistent entropy can instead be used as the internal variable parametrising strain hardening phenomena.

In the absence of external sources, the net increase in entropy of a system can be attributed to increasing internal disorder. It is mentioned that in [4] entropy was successfully used in place of an additional empirical history parameter to describe damage mechanics. For solid materials subjected to loads which result in inelastic deformations it is well known that the observed macroscopic behaviours for strain-hardening materials relate to changes in the microstructure; specifically changes in the number density and motion of dislocations. It is natural to assume then that since changes in the dislocation structure in a solid will contribute to the entropy production, this entropy can be used in a continuum model to parametrise the work-hardening behaviour. As a preliminary, incorporation of the microscopic inelastic dynamics within a continuum framework is possible by using statistical plasticity based upon ensemble averages of the dislocation kinetics in the construction of constitutive models for inelastic deformations. Thus constitutive models employ only continuum parameters such as invariants of stress and one-dimensional plastic strain, yet reflect microscopic changes in response to macroscopic disturbances. Much of the dislocation theory is now well established and is attributed to the early efforts of Orowan [28], Taylor [32] and Gillis and Gilman [9–12]. It is this physically consistent theory that forms the basis of successful continuum plasticity models [35]. It remains to quantify some function of the entropy to substitute plastic strain in parametrising the strain-hardening behaviour through changes in the mobile dislocation